RESTRICTED CIRCULATION

This paper is not an official document; it is for information only and reference cannot be made to it in any printed publication without the permission of the author

AGRICULTURAL MEMORANDUM NO 328

THE MULTI-SPAN GLASS-HOUSE

A PROCEDURE FOR ESTIMATING THE SOLAR RADIATION THAT FALLS ON THE FLOOR OF A MULTI-SPAN GLASS-HOUSE.

by C V SMITH
Meteorological Office
Cambridge
June 1970

1. INTRODUCTION

The memorandum is an attempt to set down the basis for a computer specification for the calculation. It is a natural extension of earlier work and should be read in conjunction with a previous memorandum (Ag Mem 277 - "A contribution to glasshouse design and location).

Ag Mem 277 gives a procedure for calculating the solar radiation falling on a plane surface of any slope and orientation (azimuth) at ground level. We now take this much as given and become primarily concerned with the transmission characteristics and geometry of the multispan house.

We again treat each surface of the house separately, both for direct and diffuse radiation. We note, however, that the earlier results for the single span house show that the contribution of the diffuse radiation that reaches the surfaces of the house, after "reflection" from the surrounding ground, gives rise to only a few per cent of the total radiation (direct plus diffuse) that eventually reaches the floor of the house. In the multispan house, this element of incoming radiation is no longer calculated explicity.

An additional element to be considered for the multi-span, as opposed to the single-span house is the possibility of the reflection of incoming radiation between opposing surfaces of the house.

2. HOUSE DIMENSIONS

We take our building to consist of (N) roof surfaces [(N/2) spans, each span of width (w)]. The roof surfaces are labelled 1 to N consecutively, and retain this numbering, independently of the relative position of the surface and the sun. We adopt the convention that the most southerly roof surface is that labelled (1). For a roof azimuth of 90° (by previous convention all measurements are taken from due South), the roof labelled (1) is the most westerly roof surface.

As before, we take:-

the length of the house to be (l)
the height to the eaves to be (h)
the height to the ridge to be (h')
the roofs to be symmetrical and the
roof slope to be (E)

In allowing for structural members, the ridge, gutter and purlins are taken to be of rectangular dimensions.

3. DIFFUSE RADIATION

The daily totals of diffuse sky radiation that reach the ground within the house (on the day adopted as representative of a given month) are:-

$$\sum_{\substack{\text{All}\\ \text{Surfaces}}} \sum_{t\,=\,\text{Sunrise}}^{t\,=\,\text{Sunset}} \left[RD_\gamma + \rho'(R)(I_\gamma \text{Sin}\,\gamma + D_\gamma) \right] \text{Cos}^2 E/2.60.\tau.a.P.F$$

Where I_γ = direct solar radiation intensity at the earth's surface, on a plane normal to the solar beam (cals./min.)

D_γ = diffuse radiation intensity (from clear skies)

= solar elevation (measured from a horizontal surface)

E = tilt of the receiving plane to the horizontal

R = mean relative duration of sunshine (number of hours of sunshine/day length)

$f'(R)$ = a constant of proportionality when R is determined from monthly mean values of cloudiness

τ = transmission coefficient of glass (for diffuse radiation).

a = overall area of transmitting surface

P = proportion of that surface that is transparent

= (1 - s/a), and here (s) = "area" of solid structural members within the overall area (a)

F = Angle Factor between a surface and the floor

= proportion of the radiation transmitted by a given house surface that is intercepted by the floor of the house

t = hour angle of the sun. (As before, daily sums are obtained from hourly sums. Hourly sums are obtained by assuming a representative value for the hour is given by the calculation made on the half hour)

It will be apparent that the basic problem in the present note is the evaluation of the terms (τ), (P), (F). In Ag. Mem. 277, the complexity of a rigorous treatment was hinted at, and first approximations were made to obtain numerical answers. We now treat these quantities (τ), (P), (F) more fully, since the values assigned to them will largely determine the results we shall obtain.

4. EVALUATION OF THE PROPORTION OF TRANSMITTED DIFFUSE RADIATION INTERCEPTED BY THE FLOOR OF THE HOUSE

4.1 ANGLE FACTOR ALGEBRA

Consider, as in Fig. 1, the diffuse radiation transmitted from the surface A_1, to the surface A_4.

Let $F_{A1 \longrightarrow A4}$ denote the Angle Factor

between these two surfaces (in the required sense) [and in general let the Angle Factor between any two surfaces A_n and A_m be $F_{An \longrightarrow Am}$]

In Fig. 1, we denote the intervening areas A_2, A_3 as indicated.

We write the area A_1 + area A_2 as A_{12}

the area A_3 + area A_4 as A_{34}

If the surfaces are uniform transmitters and receivers, an energy balance equation enables us to write:-

$$A_1 F_{A1 \longrightarrow A4} = A_{12} F_{A12 \longrightarrow A4} - A_2 F_{A2 \longrightarrow A4}$$

Since neither of the Angle Factors on the RHS of this expression is a commonly tabulated quantity, we proceed with a further application of the energy balance idea, and write:-

$$A_{12} F_{A12 \longrightarrow A4} = A_{12} F_{A12 \longrightarrow A34} - A_{12} F_{A12 \longrightarrow A3}$$

$$A_2 F_{A2 \longrightarrow A4} = A_2 F_{A2 \longrightarrow A34} - A_2 F_{A2 \longrightarrow A3}$$

On substitution in the above we have:-

$$\boxed{A_1 F_{A1 \longrightarrow A4} = A_{12} F_{A12 \longrightarrow A34} + A_2 F_{A2 \longrightarrow A3} - A_{12} F_{A12 \longrightarrow A3} - A_2 F_{A2 \longrightarrow A34}}$$

4.2 THE ANGLE FACTOR - THE GENERAL EXPRESSION

The general expression for the evaluation of the Angle Factor appropriate to our particular problem is known and is reproduced below.

In the general case, shown in Fig 2

the surface A_n has dimensions b, c

the surface A_m has dimensions b, g

the angle of separation is λ

$$\text{Let } X = g/b$$

$$Y = c/b$$

$$Z = X^2 + Y^2 - 2XY \cos \lambda$$

Then we may write:-

$$F_{A1 \longrightarrow A2} (\pi Y)$$

$$= - \frac{\sin 2\lambda}{4} \Big[XY \sin\lambda + \left(\frac{\pi}{2} - \lambda\right)(X^2 + Y^2) + Y^2 \tan^{-1}\left(\frac{X - Y\cos\lambda}{Y \sin\lambda}\right) + X^2 \tan^{-1}\left(\frac{Y - X\cos\lambda}{X \sin\lambda}\right) \Big]$$

$$+ \frac{\sin^2\lambda}{4} \left(\left(\frac{2}{\sin^2\lambda} - 1\right) \ln \left[\frac{(1+X^2)(1+Y^2)}{1 + Z}\right] + Y^2 \ln \left[\frac{Y^2(1+Z)}{(1+Y^2)Z}\right] + X^2 \ln \left[\frac{X^2(1+X^2)^{\cos 2\lambda}}{Z(1+Z)^{\cos 2\lambda}}\right] \right)$$

$$+ Y \tan^{-1}\left(\frac{1}{Y}\right) + X \tan^{-1}\left(\frac{1}{X}\right) - \sqrt{Z} \tan^{-1}\left(\frac{1}{\sqrt{Z}}\right)$$

$$+ \frac{\sin\lambda \sin 2\lambda}{2} \left(X \sqrt{1 + X^2 \sin^2\lambda}\right) \left[\tan^{-1}\left(\frac{X\cos\lambda}{\sqrt{1+X^2\sin^2\lambda}}\right) + \tan^{-1}\left(\frac{Y - X\cos\lambda}{\sqrt{1+X^2\sin^2\lambda}}\right) \right]$$

$$+ \cos\lambda \int_0^Y \sqrt{1+\mu^2 \sin^2\lambda} \left[\tan^{-1}\left(\frac{X - \mu\cos\lambda}{\sqrt{1+\mu^2\sin^2\lambda}}\right) + \tan^{-1}\left(\frac{\mu\cos\lambda}{\sqrt{1+\mu^2\sin^2\lambda}}\right) \right] d\mu$$

4.3 ANGLE FACTOR FOR DIFFUSE RADIATION TRANSMITTED BETWEEN A SIDE WALL AND THE FLOOR OF THE MULTISPAN HOUSE

For this we make substitution in the general expression of section 4.2 of

$$\lambda = \pi/2$$

and $\cos\lambda = 0$; $\sin\lambda = 1$

$\cos 2\lambda = -1$; $\sin 2\lambda = 0$

Now, $F_{An \longrightarrow Am}$ is given by

$$F_{An \longrightarrow Am}\ (\pi Y)$$

$$= \frac{1}{4}\left(\ln\left[\frac{(1+X^2)(1+Y^2)}{1+Z}\right] + Y^2 \ln\left[\frac{Y^2(1+Z)}{(1+Y^2)Z}\right] + X^2 \ln\left[\frac{X^2(1+X^2)^{-1}}{Z(1+Z)^{-1}}\right]\right)$$

$$+ Y\tan^{-1}(1/Y) + X\tan^{-1}(1/X) - \sqrt{Z}\tan^{-1}(1/\sqrt{Z})$$

FOR A SIDE WALL

$g = N_W/2$ and $X = N_W/2l$

$b = l$ $Y = h/l$

$c = h$ $Z = \frac{N^2 w^2}{4l^2} + \frac{h^2}{l^2}$

FOR AN END WALK

$g = l$ and $X = 2l/N_W$

$b = N_W/2$ $Y = (h+h')N_W$

$c = (h+h')/2$ $Z = \frac{4l^2}{N^2 W^2} + \frac{(h+h')^2}{N^2 w^2}$

Where we have again made the assumption of Ag. Mem 277 that the end wall may be regarded as being of rectangular dimensions with a mean height of $(h+h')/2$.

4.4 ANGLE FACTOR FOR DIFFUSE RADIATION TRANSMITTED FROM A ROOF SURFACE TO THE FLOOR OF THE HOUSE

We consider at this stage only the diffuse radiation from a roof surface that reaches the floor of the house directly, ie without traversing or reflection from any other surface of the house. The floor area we assume as receiving diffuse radiation from a given roof surface is then limited by the requirement that any point on it should be able to view all parts of the transmitting roof surface directly. Fig. 3 (in which the notation has been carried over from Fig. 1) shows, in section, the relative disposition of the transmitting roof and receiving floor surfaces.

Consider as in Fig. 4, a multispan house with (N) roof surfaces. The width of the floor area that intercepts diffuse radiation directly from the (n^{th}) roof surface is denoted by A_{fn}. Its length is of course (1).

The magnitude of A_{fn} may be obtained from one of the following expressions:-

$n = 1$ or N

Case I If $h'\cot E \leq (N-1)\ w/2$; $A_{f1} = A_{fN} = w/2 + h'\cot E$

Case II If $h'\cot E > (N-1)\ w/2$; $A_{f1} = A_{fn} = Nw/2$

$n \neq 1$ or N

n is an odd number

Case III $\left.\begin{array}{l} \text{If } h'\cot E \leq nw/2 \\ \text{and } h'\text{cote } E \leq (N-n)w/2 \end{array}\right\}$; $A_{fn} = 2\ h'\cot E$

Case IV $\left.\begin{array}{l} \text{If } h'\cot E > nw/2 \\ \text{and } h'\cot E \leq (N-n)\ w/2 \end{array}\right\}$; $A_{fn} = nw/2 + h'\cot E$

Case V $\left.\begin{array}{l} \text{If } h'\cot E \leq nw/2 \\ \text{and } h'\cot E > (N-n)w/2 \end{array}\right\}$; $A_{fn} = h'\cot E + (N-n)w/2$

Case VI $\left.\begin{array}{l} \text{If } h'\cot E > nw/2 \\ \text{and } h'\cot E > (N-n)w/2 \end{array}\right\}$; $A_{fn} = Nw/2$

n is an even number

Case VII $\left.\begin{array}{l} \text{If } h'\cot E \leq (n-1)w/2 \\ \text{and } h'\cot E \leq (N-n+1)w/2 \end{array}\right\}$; $A_{fn} = 2\ h'\cot E$

Case VIII $\left.\begin{array}{l} \text{If } h'\cot E > (n-1)w/2 \\ \text{and } h'\cot E \leq (N-n+1)w/2 \end{array}\right\}$; $A_{fn} = (n-1)w/2 + h'\cot E$

Case IX $\left.\begin{array}{l} \text{If } h'\cot E \leq (n-1)W/2 \\ h'\cot E > (N-n+1)w/2 \end{array}\right\}$; $A_{fn} = h'\cot E + (N-n+1)w/2$

Case X $\left.\begin{array}{l} \text{If } h'\cot E > (n-1)w/2 \\ h'\cot E > (N-n+1)W/2 \end{array}\right\}$; $A_{fn} = Nw/2$

A flow diagram for arriving at the appropriate value of A_{fn} is set out on a separate sheet.

The alternative configurations of roof surface and floor area of Cases I to X are set out schematically in Fig. 5.

It would appear from Fig. 5 that in the application of Angle Factor algebra to the evaluation of (F), that Cases III, V, VII, VIII fall within a common pattern, whilst Cases I, II, IV, VI, IX, X form a second distinct group.

In the first group, the floor area of interest and the extension of the radiating roof surface have a common edge. In the second group, both the radiating roof surface and the floor area of interest have to be extended to ~~achieve~~ a common edge. In the notation of Fig. 1 and Section 4.1 the roof surface A_1 and the floor area A_{34} represent the situation in the first group. The second group are represented by the situation in which the roof surface is A_1 and the floor area is A_4.

4.4.1 CALCULATION OF ROOF-FLOOR ANGLE FACTORS (CASE III, ETC.).

Using the notation of section 4.1 and Fig. 1, a general expression for $F_{A1 \rightarrow A34}$ is given by:-

$$A_1 F_{A1 \rightarrow A34} = A_{12} F_{A12 \rightarrow A34} - A_2 F_{A2 \rightarrow A34}$$

or

$$\boxed{F_{A1 \rightarrow A34} = \frac{A_{12}}{A_1} F_{A12 \rightarrow A34} \quad \frac{A_2}{A_1} F_{A2 \rightarrow A34}}$$

Case III

The angle between the surfaces A_{12}, A_{34} $= \lambda = E =$ roofslope

$A_1 = l\,(h'-h)$ cosec E

$A_2 = l\,h$ cosec E

$A_{12} = l\,h'$ cosec E

$A_{34} = l A_{fn} = 2\,l\,h'$ cot E

From which $A_{12}/A_1 = h'/(h'-h)$

$A_2/A_1 = h/(h'-h)$

EVALUATION OF $F_{A12 \longrightarrow A34}$

In the notation of Fig. 2

$g = 2h' \cot E$ $\qquad$ $X = (2\ h' \cot E)/1$

$b = 1$ $\qquad$ $Y = (h' \operatorname{cosec} E)/\ell$

$c = h' \operatorname{cosec} E$ $\qquad$ $Z = [(h' \operatorname{cosec} E)/1]^2$

The general expression of Section 4.2 can now be evaluated.

EVALUATION OF $F_{A2 \longrightarrow A34}$

In the notation of Fig. 2.

$g = 2\ h' \cot E$ $\qquad$ $X = (2\ h' \cot E)/1$

$b = 1$ $\qquad$ $Y = (h \operatorname{cosec} E)/1$

$c = h \operatorname{cosec} E$

$$Z = \left(\frac{2\ h' \cot E}{1}\right)^2 + \left(\frac{h \operatorname{cosec} E}{1}\right)^2 - 2\left(\frac{2\ h' \cot E}{1}\right)\left(\frac{h \operatorname{cosec} E}{1}\right) \operatorname{Cos} E$$

$$Z = \left(\frac{2\ h' \cot E}{1}\right)^2 + \left(\frac{h \operatorname{cosec} E}{1}\right)^2 - h'h\left(\frac{2 \cot E}{1}\right)^2$$

Substitution may now be made in the general expression of Section 4.2.

Case VII

$F_{A1 \longrightarrow A34}$ in Case VII is numerically equal to the value for $F_{A1 \longrightarrow A34}$ that would be obtained for Case III.

Case V

The angle between the surfaces A_{12}, $A_{34} = \lambda = E$

$A_1 = 1\ (h'-h) \operatorname{cosec} E$

$A_2 = 1\ h \operatorname{cosec} E$

$A_{12} = 1\ h' \operatorname{cosec} E$

$A_{34} = 1A_{fn} = 1\ [h' \cot E + (N-n)\ w/2]$

$A_{12}/A_1 = h'/(h'-h)$

$A_2/A_1 = h/(h'-h)$

EVALUATION OF $F_{A12 \longrightarrow A34}$

In the notation of Fig. 2.

$g = [h' \cot E + (N-n)w/2]$

$b = 1$

$c = h' \operatorname{cosec} E$

$X = [h' \cot E + (N-n)\, w/2]/1$

$Y = [h' \operatorname{cosec} E]/1$

$$Z = \left[\frac{h' \cot E + (N-n)w/2}{1}\right]^2 + \left[\frac{h' \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2h' \cot E}{1}\right]\left[\frac{h' \cot E + (N-n)w/2}{1}\right]$$

The general expression of section 4.2 can now be evaluated.

EVALUATION OF $F_{A2 \longrightarrow A34}$

In the notation of Fig. 2

$g = [h' \cot E + (N-n)w/2]$

$b = 1$

$c = h \operatorname{cosec} E$

$X = [h' \cot E + (N-n)w/2]/1$

$Y = [h \operatorname{cosec} E]/1$

$$Z = \left[\frac{h' \cot E + (N-n)w/2}{1}\right]^2 + \left[\frac{h \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2\, h' \cot E}{1}\right]\left[\frac{h' \cot E + (N-n)w/2}{1}\right]$$

The general expression of section 4.2 can now be evaluated.

Case VIII

As before

$A_{12}/A_1 \quad = \quad h'/(h'-h)$

$A_2/A_1 \quad = \quad h/(h'-h)$

EVALUATION OF $F_{A12 \longrightarrow A34}$

In the notation of Fig. 2

$$g = h' \cot E + (n-1)w/2$$

$$b = 1$$

$$c = h' \operatorname{cosec} E$$

$$X = [h' \cot E + (n-1)w/2]/1$$

$$Y = [h' \operatorname{cosec} E]/1$$

$$Z = \left[\frac{h' \cot E + (n-1)w/2}{1}\right]^2 + \left[\frac{h' \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2\, h' \cot E}{1}\right]\left[\frac{h' \cot E + (n-1)w/2}{1}\right]$$

EVALUATION OF $F_{A2 \longrightarrow A34}$

In the notation of Fig. 2

$$g = h' \cot E + (n-1)w/2$$

$$b = 1$$

$$c = h \operatorname{cosec} E$$

$$X = [h' \cot E + (n-1)w/2]/1$$

$$Y = [h \operatorname{cosec} E]/1$$

$$Z = \left[\frac{h' \cot E + (n-1)w/2}{1}\right]^2 + \left[\frac{h \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2\, h \cot E}{1}\right]\left[\frac{h' \cot E + (n-1)w/2}{1}\right]$$

Substitution of X, Y, Z has now to be made in the general expression of Section 4.2.

4.4.2 CALCULATION OF ROOF-FLOOR ANGLE FACTORS (CASE I, ETC.)

The appropriate Angle Factor algebra has been set out in Section 4.1 and Fig. 1 but is reproduced again here.

$$\boxed{F_{A1 \longrightarrow A4} = \frac{A_{12}}{A_1} . F_{A12 \longrightarrow A34} + \frac{A_2}{A_1} . F_{A2 \longrightarrow A3} - \frac{A_{12}}{A_1} . F_{A12 \longrightarrow A3} - \frac{A_2}{A_1} . F_{A2 \longrightarrow A34}}$$

Case I

$A_1 = 1(h'-h) \text{ cosec } E$ $\qquad A_{12} = 1 h' \text{ cosec } E$

$A_2 = 1 h \text{ cosec } E$

$A_3 = 1 [h' \cot E - w/2]$

$A_4 = 1 [h' \cot E + w/2]$

$A_{12}/A_1 = h'/(h'-h)$; $\qquad A_2/A_1 = h/(h'-h)$

EVALUATION OF $F_{A12 \longrightarrow A34}$

In the notation of Fig. 2

$g = 2 h' \cot E$ $\qquad X = (2h' \cot E)/1$

$b = 1$ $\qquad Y = (h' \text{ cosec } E)/1$

$c = h' \text{cosec } E$ $\qquad Z = [(h' \text{ cosec } E)/1]^2$

EVALUATION OF $F_{A2 \longrightarrow A3}$

In the notation of Fig. 2

$g = [h' \cot E - w/2]$

$b = 1$

$c = h \text{ cosec } E$

$X = [h' \cot E - w/2]/ 1$

$Y = (h \text{ cosec } E)/1$

$$Z = \left[\frac{h' \cot E - w/2}{1}\right]^2 + \left[\frac{h \text{ cosec } E}{1}\right]^2 - \left[\frac{2 h \cot E}{1}\right]\left[\frac{h' \cot E - w/2}{1}\right]$$

EVALUATION OF $F_{A12 \longrightarrow A3}$

In the notation of Fig. 2

$g = [h' \cot E - w/2]$

$b = 1$

$c = h' \text{ cosec } E$

$$X = [h' \cot E - w/2]/l$$

$$Y = (h' \operatorname{cosec} E)/l$$

$$Z = \left[\frac{h' \cot E - w/2}{l}\right]^2 + \left[\frac{h' \operatorname{cosec} E}{l}\right]^2 - \left[\frac{2\,h \cot E}{l}\right]\left[\frac{h' \cot E - w/2}{l}\right]$$

Evaluation of F_{A2} → .34

In the notation of Fig 2

$$g = 2h' \cot E$$

$$b = l$$

$$c = h \operatorname{cosec} E$$

$$X = (2h' \cot E)/l$$

$$Y = (h \operatorname{cosec} E)/l$$

$$Z = \left[\frac{2h' \cot E}{l}\right]^2 + \left[\frac{h \operatorname{cosec} E}{l}\right]^2 - \left[\frac{2h \cot E}{l}\right]\left[\frac{2h' \cot E}{l}\right]$$

Case II

$A_1 = 1(h'-h) \operatorname{cosec} E \quad ; \quad A_{12}/A_1 = h'/(h'-h)$

$A_2 = 1 \; h \operatorname{cosec} E$

$A_{12} = 1 \; h' \operatorname{cosec} E \quad ; \quad A_2/A_1 = h/(h'-h)$

$A_3 = 1[h' \cot E - w/2]$

$A_4 = 1[Nw/2]$

$A_{34} = 1[h' \cot E + (N-1)\, w/2]$

EVALUATION OF $F_{A12 \longrightarrow A34}$

In the notation of Fig. 2

$g = [h' \cot E + (N-1)\, w/2]$

$b = 1$

$c = h' \operatorname{cosec} E$

$X = [h' \cot E + (N-1)\, w/2]/1$

$Y = [h' \operatorname{cosec} E]/1$

$$Z = \left[\frac{h' \cot E + (N-1)w/2}{1}\right]^2 + \left[\frac{h' \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2\,h' \cot E}{1}\right]\left[\frac{h' \cot E + (N-1)w/2}{1}\right]$$

EVALUATION OF $F_{A2 \longrightarrow A3}$

In the notation of Fig. 2

$g = h' \cot E - w/2$

$b = 1$

$c = h \operatorname{cosec} E$

$X = (h' \cot E - w/2)/1$

$Y = (h \operatorname{cosec} E)/1$

$$Z = \left[\frac{h' \cot E - w/2}{1}\right]^2 + \left[\frac{h \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2\,h \cot E}{1}\right]\left[\frac{h' \cot E - w/2}{1}\right]$$

EVALUATION OF $F_{A12 \longrightarrow A3}$

In the notation of Fig. 2

$$g = h' \cot E - w/2$$

$$b = l$$

$$c = h' \operatorname{cosec} E$$

$$X = (h' \cot E - w/2)/l$$

$$Y = (h' \operatorname{cosec} E)/l$$

$$Z = \left[\frac{h' \cot E - w/2}{l}\right]^{\overset{1}{2}} + \left[\frac{h' \operatorname{cosec} E}{l}\right]^{2} - \left[\frac{2h' \cot E}{l}\right]\left[\frac{h' \cot E - w/2}{l}\right]$$

EVALUATION OF $F_{A2 \longrightarrow A34}$

In the notation of Fig. 2

$$g = h' \cot E + (N-1)\, w/2$$

$$b = l$$

$$c = h \operatorname{cosec} E$$

$$X = [h' \cot E + (N-1)\, w/2]/l$$

$$Y = [h \operatorname{cosec} E]/l$$

$$Z = \left[\frac{h' \cot E + (N-1)w/2}{l}\right]^{2} + \left[\frac{h \operatorname{cosec} E}{l}\right]^{2} - \left[\frac{2\ h \cot E}{l}\right]\left[\frac{h' \cot E + (N-1)w/2}{l}\right]$$

Case IV

$A_1 = l\ (h'-h) \operatorname{cosec} E$; $A_{12}/A_1 = h'/(h'-h)$

$A_2 = l\ (h \operatorname{cosec} E)$

$A_{12} = l\ h' \operatorname{cosec} E$; $A_2/A_1 = h/(h'-h)$

$A_3 = l\ (h' \cot E - nw/2)$

$A_4 = l\ (h' \cot E + nw/2)$

$A_{34} = l\ (2\ h' \cot E)$

EVALUATION OF $F_{A12 \longrightarrow A34}$

In the notation of Fig. 2

$$g = 2\,h' \cot E$$

$$b = 1$$

$$c = h' \operatorname{cosec} E$$

$$X = (2\,h' \cot E)/1$$

$$Y = (h' \operatorname{cosec} E)/1$$

$$Z = \left[(h' \operatorname{cosec} E)/1\right]^2$$

EVALUATION OF $F_{A2 \longrightarrow A3}$

In the notation of Fig. 2

$$g = (h' \cot E - nw/2)$$

$$b = 1$$

$$c = h \operatorname{cosec} E$$

$$X = (h' \cot E - nw/2)/1$$

$$Y = (h \operatorname{cosec} E)\ 1$$

$$Z = \left[\frac{h' \cot E - nw/2}{1}\right]^2 + \left[\frac{h \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2\,h \cot E}{1}\right]\left[\frac{h' \cot E - nw/2}{1}\right]$$

EVALUATION OF $F_{A12 \longrightarrow A3}$

In the notation of Fig. 2

$$g = h' \cot E - nw/2$$

$$b = 1$$

$$c = h' \operatorname{cosec} E$$

$$X = (h' \cot E - n\,w/2)/1$$

$$Y = (h' \operatorname{cosec} E)/1$$

$$Z = \left[\frac{h' \cot E - n\,w/2}{1}\right]^2 + \left[\frac{h' \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2h' \cot E}{1}\right]\left[\frac{h' \cot E - n\,w/2}{1}\right]$$

EVALUATION OF $F_{A2 \longrightarrow A34}$

In the notation of Fig. 2

$$g = 2\,h' \cot E$$

$$b = 1$$

$$c = h \operatorname{cosec} E$$

$$X = (2\,h' \cot E)/1$$

$$Y = (h \operatorname{cosec} E)/1$$

$$Z = \left[\frac{2h' \cot E}{1}\right]^2 + \left[\frac{h \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2\,h \cot E}{1}\right]\left[\frac{2\,h' \cot E}{1}\right]$$

$$Z = h'(h'-h)\left[\frac{2 \cot E}{1}\right]^2 + \left[\frac{h \operatorname{cosec} E}{1}\right]^2$$

Case VI

$$A_1 = 1\,(h'-h) \operatorname{cosec} E \qquad ; \quad A_{12}/A_1 = h'/(h'-h)$$

$$A_2 = 1\,(h \operatorname{cosec} E)$$

$$A_{12} = 1\,h' \operatorname{cosec} E \qquad ; \quad A_2/A_1 = h/(h'-h)$$

$$A_3 = 1(h' \cot E - nw/2)$$

$$A_4 = 1\,(N)\,w/2$$

$$A_{34} = 1\,[h' \cot E + (N-n)\,w/2]$$

EVALUATION OF $F_{A12 \longrightarrow A\,34}$

In the notation of Fig. 2

$$g = h' \cot E + (N-n)\,w/2$$

$$b = 1$$

$$c = h' \operatorname{cosec} E$$

$$X = [h' \cot E + (N-n)\,w/2]/1$$

$$Y = [h' \operatorname{cosec} E]/1$$

$$Z = \left[\frac{h' \cot E + (N-n)w/2}{1}\right]^2 + \left[\frac{h' \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2h' \cot E}{1}\right]\left[\frac{h' \cot E + (N-n)w/2}{1}\right]$$

EVALUATION OF $F_{A2 \longrightarrow A3}$

In the notation of Fig. 2

$$g = h' \cot E - n_w/2$$

$$b = 1$$

$$c = h \operatorname{cosec} E$$

$$X = \left[h' \cot E - n_w/2\right]/1$$

$$Y = \left[h \operatorname{cosec} E\right]/1$$

$$Z = \left[\frac{h' \cot E - n_w/2}{1}\right]^2 + \left[\frac{h \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2h \cot E}{1}\right]\left[\frac{h' \cot E - n_w/2}{1}\right]$$

EVALUATION OF $F_{A12 \longrightarrow A3}$

In the notation of Fig. 2

$$g = h' \cot E - nw/2$$

$$b = 1$$

$$c = h' \operatorname{cosec} E$$

$$X = (h' \cot E - nw/2)/1$$

$$Y = (h' \operatorname{cosec} E)/1$$

$$Z = \left[\frac{h' \cot E - nw/2}{1}\right]^2 + \left[\frac{h' \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2\,h' \cot E}{1}\right]\left[\frac{h' \cot E - n_w/2}{1}\right]$$

EVALUATION of $F_{A2 \longrightarrow A34}$

$$g = h' \cot E + (N-n)\,w/2$$

$$b = 1$$

$$c = h \operatorname{cosec} E$$

$$X = \left[h' \cot E + (N-n)\,w/2\right]/1$$

$$Y = \left[h \operatorname{cosec} E\right]/1$$

$$Z = \left[\frac{h' \cot E + (N-n)w/2}{1}\right]^2 + \left[\frac{h \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2\,h \cot E}{1}\right]\left[\frac{h' \cot E + (N-n)w/2}{1}\right]$$

Case IX

$$A_1 = l(h'-h)\ \mathrm{cosec}\ E \quad ; \quad A_{12}/A_1 = h'/(h'-h)$$

$$A_2 = l(h\ \mathrm{cosec}\ E)$$

$$A_{12} = l(h'\ \mathrm{cosec}\ E) \quad ; \quad A_2/A_1 = h/(h'-h)$$

$$A_3 = l[h'\cot E - (N-n+1)\,w/2]$$

$$A_4 = l[h'\cot E + (N-n+1)\,w/2]$$

$$A_{34} = l[2\,h'\cot E]$$

EVALUATION OF $F_{A12 \longrightarrow A34}$

In the notation of Fig. 2

$$g = 2\,h'\cot E$$

$$b = l$$

$$c = h'\ \mathrm{cosec}\ E$$

$$X = (2\,h'\cot E)/l$$

$$Y = (h'\ \mathrm{cosec}\ E)/l$$

$$Z = [(h'\ \mathrm{cosec}\ E)/l]^2$$

EVALUATION OF $F_{A2 \longrightarrow A3}$

In the rotation of Fig. 2

$$g = h'\cot E - (N-n+1)\,w/2$$

$$b = l$$

$$c = h\ \mathrm{cosec}\ E$$

$$X = [h'\cot E - (N-n+1)\,w/2]/l$$

$$Y = [h\ \mathrm{cosec}\ E]/l$$

$$Z = \left[\frac{h'\cot E-(N-n+1)w/2}{l}\right]^2 + \left[\frac{h\ \mathrm{cosec}\ E}{l}\right]^2 - \left[\frac{2\,h\cot E}{l}\right]\left[\frac{h'\cot E-(N-n+1)w/2}{l}\right]$$

EVALUATION OF $F_{A12 \longrightarrow A3}$

In the notation of Fig. 2

$$g = h' \cot E - (N-n+1)w/2$$

$$b = 1$$

$$c = h' \operatorname{cosec} E$$

$$X = [h' \cot E - (N-n+1)\, w/2]/1$$

$$Y = [h' \operatorname{cosec} E]/1$$

$$Z = \left[\frac{h' \cot E - (N-n+1)w/2}{1}\right]^2 + \left[\frac{h' \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2h' \cot E}{1}\right]\left[\frac{h' \cot E - (N-n+1)w/2}{1}\right]$$

EVALUATION OF $F_{A2 \longrightarrow A34}$

In the notation of Fig. 2

$$g = 2\, h' \cot E$$

$$b = 1$$

$$c = h \operatorname{cosec} E$$

$$X = (2\, h' \cot E)/1$$

$$Y = (h \operatorname{cosec} E)/1$$

$$Z = \left[\frac{2h' \cot E}{1}\right]^2 + \left[\frac{h \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2h \cot E}{1}\right]\left[\frac{2\, h' \cot E}{1}\right]$$

$$Z = h'(h'-h)\left[\frac{2 \cot E}{1}\right]^2 + \left[\frac{h \operatorname{cosec} E}{1}\right]^2$$

CASE X

$A_1 = \ell(h^1-h)$ cosec E; $\qquad A_{12}/A_1 = h^1/(h^1-h)$

$A_2 = \ell(h$ cosec E);

$A_{12} = \ell(h^1$ cosec E); $\qquad A_2/A_1 = h/(h^1-h)$

$A_2 = \ell[h^1 \cot E - (N-n+1)w/2]$

$A_4 = \ell[N w/2]$

$A_{34} = \ell[h^1 \cot E + (N-1)W/2]$

EVALUATION OF $F_{A12 \rightarrow A34}$

In the notation of Fig 2

$g = h^1 \cot E + (n-1)w/2$

$b = l$

$c = h^1 \operatorname{cosec} E$

$X = [h^1 \cot E + (n-1)w/2]/l$

$Y = [h^1 \operatorname{cosec} E]/\ell$

$$Z = \left[\frac{h^1 \cot E + (n-1)w/2}{l}\right]^2 + \left[\frac{h^1 \operatorname{cosec} E}{l}\right]^2 + \left[\frac{2h^1 \cot E}{l}\right]\left[\frac{h^1 \cot E + (n-1)W/2}{l}\right]$$

EVALUATION OF $F_{A2 \rightarrow A3}$

In the notation of Fig 2

$g = h^1 \cot E - (N-n+1)w/2$

$b = l$

$c = h \operatorname{cosec} E$

$X = [h^1 \cot E - (N-n+1)w/2]/l$

$Y = [h \operatorname{cosec} E]/l$

$$X = \left[\frac{h^1 \cot E - (N-n+1)w/2}{l}\right]^2 + \left[\frac{h \operatorname{cosec} E}{l}\right]^2 - \left[\frac{2h \cot E}{l}\right]\left[\frac{h^1 \cot E - (N-n+1)w/2}{l}\right]$$

EVALUATION OF $F_{A12 \quad A3}$

In the notation of Fig 2

$g = h^1 \cot E - (N-n+1)W/2$

$b = l$

$c = h^1$ cosec

$X = [h^1 \cot E - (N-n+1)W/2]/l$

$$Y = \left[h^1 \operatorname{cosec} E\right]/1$$

$$Z = \left[\frac{h^1 \cot E - (N-n+1)w/2}{1}\right]^2 + \left[\frac{h^1 \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2h^1 \cot E}{1}\right]\left[\frac{h^1 \cot E-(N-n+1)w/2}{1}\right]$$

EVALUATION OF $F_{A2 \quad A34}$

In the notation of Fig 2

$$g = h^1 \cot E + (n-1)w/2$$

$$b = 1$$

$$c = h \operatorname{cosec} E$$

$$X = \left[h^1 \cot E + (n-1)w/2\right]/1$$

$$Y = \left[h \operatorname{cosec} E\right]/1$$

$$Z = \left[\frac{h^1 \cot E + (n-1)w/2}{1}\right]^2 + \left[\frac{h \operatorname{cosec} E}{1}\right]^2 - \left[\frac{2h \cot E}{1}\right]\left[\frac{h^1 \cot E + (n-1)w/2}{1}\right]$$

5. Augmentation (by reflection) of diffuse radiation that reaches the floor of the house

5.1. "Reflection" between external roof surfaces

The transmission coefficient (Υ) of our glass for diffuse radiation is of the order 0.7 to 0.8. Ignoring absorbtion by the glass, a proportion, ($1-\Upsilon$) of the incident diffuse radiation is scattered back to the sky and part of this "reflected" radiation will be captured by opposing roof surfaces. In other words, the multi-span roof structure leads to an augmented diffuse sky radiation component entering the house.

This argument does not apply to the two end roof surfaces ($n = 1$ and N); but since we have chosen to omit calculation of the diffuse sky radiation scattered back from the ground surrounding the house, we propose, for the sake of simplicity and uniformity, to augment the diffuse sky radiation reaching each and every roof surface by the same amount.

If the diffuse sky radiation arriving per unit area per unit time on our roof surface is D_s,

$$\text{where } D_s = \left(RD_\gamma + f'(R) \left[\bar{I}_\gamma \sin \gamma + D_\gamma \right] \right)$$

the diffuse radiation scattered back to the sky is $D_s \, (1 - \Upsilon)$

The amount by which D_s is augmented due to back scattering between opposing roof surfaces we take to be $D_s \, (1 - \Upsilon) \, F_s$

where F_s = Angle Factor for diffuse radiation between two opposing roof surfaces.

(F_s) is calculated from the general expression of section 4.2. Using the notation of Fig. 2 we have

$$g = (h'-h) \operatorname{cosec} E$$

$$b = 1$$

$$c = (h'-h) \operatorname{cosec} E$$

$$\lambda = (\pi - 2E)$$

$$X = Y = \left[(h'-h) \operatorname{cosec} E \right] / 1$$

$$Z = 2\left[\frac{(h'-h)\ \mathrm{cosec}\ E}{1}\right]^2 - 2\left[\frac{(h'-h)\ \mathrm{cosec}\ E}{1}\right]^2 \cos(\pi - 2E)$$

$$Z = 2\left[\frac{(h'-h)\ \mathrm{cosec}\ E}{1}\right]^2 (1 + \cos 2E)$$

5.2 "Reflection" between internal roof surfaces

It is readily apparent that the Angle Factor for diffuse radiation between opposing internal roof surfaces, is numerically the same as that for opposing external roof surfaces.

The augmented diffuse sky radiation reaching any external roof surface is of intensity

$$D_s + D_s(1-\tau)F_s$$

The corresponding transmitted diffuse sky radiation intensity is

$$(\tau)\left[Ds + Ds(1-\tau)F_s\right]$$

$$= (\tau)\ Ds\left[1 + (1-\tau)F_s\right] = D(\text{say})$$

The proportion of the diffuse radiation emitted by an internal roof surface that is captured by an opposing roof surface is (F_s).

The proportion of this intercepted radiation that is scattered back into the house is $(1-\tau)$.

The effective intensity of the diffuse radiation emanating from an internal roof surface we take as $D\left[1 + (1-\tau)Fs\right]$

The intensity of diffuse radiation emitted from an internal roof surface (permitting augmentation by one external and one internal reflection) is thus

$$(\tau)\ D_s\left[1 + (1-\tau)F_s\right]^2$$

and is equivalent to an augmented diffuse sky radiation of

$$Ds\left[1 + (1-\tau)F_s\right]^2$$

Since for the walls we have so far omitted consideration of scattered diffuse radiation arriving from the ground outside the house, and also the diffuse radiation arriving at the walls from all other internal surfaces, we made the assumption that, for the sake of simplicity, the diffuse sky radiation reaching all external surfaces may be written as

$$Ds\left[1 + (1-\tau)F_s\right]^2$$

The daily totals of diffuse sky radiation that reach the ground within the house are now to be calculated as

$$\sum_{\substack{\text{all}\\\text{surfaces}}} \sum_{t\,=\,\text{sunrise}}^{t\,=\,\text{sunset}} \left[1 + (1-\tau)F_s\right]^2 \left[\bar{R}D_\gamma + f'\,(R)(I_\gamma \sin\gamma + D_\gamma)\right]\cos^2 E/2.60.\tau.a.\,\rho.F.$$

where (F) between any given surface and the ground within the house is computed by the procedure set out in sections 4.2 to 4.4.2.

6. EVALUATION OF (P) FOR DIFFUSE RADIATION

P is the proportion of the overall area of the glasshouse surface that may be regarded as freely transmitting. It takes account of the structural members in the plane of the glass and should properly take some account of the fact that the structural members will be solid, possessing depth.

Consider a small horizontal area dS beneath a uniformly radiating hemsiphere.

The amount of radiation reaching dS from a direction making an angle (β) with the normal to dS is proportional to dS $\cos\beta$.

Now consider the radiation intercepted at any one point on dS.

The amount received through the small solid angle $d\omega$ is proportional $d\omega$.

For the small solid angle between directions

$$\beta \text{ and } \beta + d\beta$$

$$\theta \text{ and } \theta + d\theta$$

(where θ specifies the radiation in azimuth)

we have, $d\omega = \sin\beta \, . d\beta \, . d\theta$

We may write the radiation intercepted by the whole area dS in time dt /the radiation having directions between (β) and ($\beta + d\beta$) and (θ) and ($\theta + d\theta$)/ as proportional to

$$dS \cos\beta \, d\omega \, dt$$

or equal to $K \, dS \cos\beta \, d\omega \, dt$

(where K is a constant)

The total amount of radiation intercepted by the area dS, from above, in time dt is obtained by integration over the hemisphere. For this, the limits are

$$\beta = 0 \text{ to } = \pi/2$$

$$\theta = 0 \text{ to } \theta = 2\pi$$

$$\text{Now } K \ dS \ dt \int_0^{2\pi} d\theta \int_0^{\pi/2} \cos\beta \sin\beta \, d\beta = \pi K \, . \, dS \, . \, dt$$

So that the radiation received by unit area of the surface dS in unit time may be written as

$$\pi K$$

If our elemental area dS is now regarded as lying in the surface of a horizontal glazing bar, of width (b) and depth (d), the equivalent surface area of unit length of the bar, for radiation incidentation angle (β), may be written as

$$dS = b + d \tan\beta$$

The total amount of radiation from above intercepted by unit length of the bar in time dt is again obtained by integration over the hemisphere and is given by

$$Kdt \int_0^{2\pi} d\theta \int_0^{\pi/2} \sin\beta \cos\beta \, (b + d \tan\beta) \, d\beta$$

$$= 2\pi K \, dt \, (b/2 + d\pi/4)$$

If the horizontal separation of the glazing bars is (c) we may write

$$P = 1 - \frac{2 K(b/2 + d\pi/4).\pi}{\pi K c}$$

or

$$\boxed{P = 1 - (2b + d\pi)/2c}$$

We note that as (d) tends to zero, P tends to (1 - b/c) which is eminently reasonable.

For a vertical wall and glazing bars, a similar derivation may be applied, when the expressions involving (β) and (θ) are interchanged. For a wall, we are concerned only with diffuse sky radiation from half the celestial vault. Whilst the numerical value of the integrals is thereby reduced by one half, the same reduction applies to both the numerator and denominator of the fraction on the RHS of the expression for (P) above, and the final expression for (P) remains unchanged.

The expression developed for (P) (for diffuse radiation) is assumed to apply to all surfaces, no matter what their tilt.

7. DIRECT RADIATION

The daily totals of direct radiation that reach the ground within the house (on the day adopted as representative of a given month) are

$$\sum_{\substack{\text{all}\\ \text{surfaces}}} \quad \sum_{t\,=\,\text{sunrise}}^{t\,=\,\text{sunset}} 60.\ I_\gamma \cos\beta .\ R.\tau.a.P.\ F.$$

where I_γ = direct solar radiation intensity at the earth's surface, on a plane normal to the solar beam (cal./min)

γ = solar elevation

β = angle of incidence of the radiation onto the plane of interest

R = mean relative duration of sunshine

τ = transmission coefficient of glass (for direct radiation at an angle β)

a = overall area of Transmitting Surface

P = proportion of that surface which is transparent

F = angle factor

= proportion of the radiation transmitted by a given house surface that is intercepted by the floor of the house.

t = hour angle of the sun (values on the half hour between sunrise and sunset).

8. DIRECT RADIATION THROUGH THE WALLS

EVALUATION OF (F) , the proportion of the transmitted radiation that is intercepted by the floor of the house.

The procedures and generalised expressions for the quantity (F) developed in the earlier memorandum (Ag Memo 277) still hold. They are reproduced again now for completeness.

We note the appropriate floor dimensions are now (l) and (Nw/2)

We recall that $\theta = t - A_p$

Where θ = solar - wall azimuth

t = hour angle of the sun

A_p = azimuth of the wall

= angle between due south and the outward normal to the wall

Side Wall - dimensions (h) and (l)

(i) $\theta = 90^\circ$ - sun at grazing incidence $\rightarrow \boxed{F = 0}$

(ii) $|\theta| > 90^\circ$ - sun impinges first on the internal surface of the wall $\rightarrow \boxed{F = 0}$

$$\text{(iii)}\ \left.\begin{array}{l} |h \cot\gamma \sin\theta| < l \\ \text{and } |h \cot\gamma \cos\theta| < Nw/2 \end{array}\right) \longrightarrow \boxed{F = 1 - |(h/2l) \cot\gamma \sin\theta|}$$

$$\text{(iv)}\ \left.\begin{array}{l} |h \cot\gamma \sin\theta| < l \\ \text{and } |h \cot\gamma \cos\theta| \geq Nw/2 \end{array}\right) \longrightarrow \boxed{F = \frac{l.Nw/2 - |\frac{1}{2} (Nw/2)^2 \tan\theta|}{|l.h \cot\gamma \cos\theta|}}$$

$$\text{(v)}\ |h \cot\gamma \sin\theta| \geq l \longrightarrow \boxed{F = \frac{l}{|2 h \cot\gamma \sin\theta|}}$$

End wall - dimensions $(h + h')/2$ and $Nw/2$

(i) $\theta = 90^\circ \rightarrow \boxed{F = 0}$

(ii) $|\theta| > 90^\circ \rightarrow \boxed{F = 0}$

$$\text{(iii)}\ \left.\begin{array}{l} \left|\left[(h + h')/2\right] \cot\gamma \sin\theta\right| < Nw/2 \\ \text{and } \left|\left[(h + h')/2\right] \cot\gamma \cos\theta\right| < l \end{array}\right\} \rightarrow \boxed{F = 1 - \left|\left(\frac{h + h'}{2Nw}\right) \cot\gamma \sin\theta\right|}$$

$$\text{(iv)}\ \left.\begin{array}{l} \left|\left[(h + h')/2\right] \cot\gamma \sin\theta\right| < Nw/2 \\ \text{and } \left|\left[(h + h')/2\right] \cot\gamma \cos\theta\right| \geq l \end{array}\right\} \rightarrow \boxed{F = \frac{l.Nw/2 - |\frac{1}{2} l^2 \tan\theta|}{\left|\left[(h + h') Nw/4\right] \cot\gamma \cos\theta\right|}}$$

$$\text{(v)}\ \left|\left[(h + h')2\right] \cot\gamma \sin\theta\right| \geq Nw/2 \rightarrow \boxed{F = \frac{(Nw/2)}{|(h + h') \cot\gamma \sin\theta|}}$$

9. DIRECT RADIATION THROUGH ROOF SURFACES

To introduce the problems involved, we consider first the simplest case, specified by the following conditions.

(i) $n = 1$ [End roof surface]

(ii) $\theta = 0$; $(A_{p1}) = o$ [Roof surface faces the sun]

(iii) All direct radiation entering through the surface falls within the floor of the house

(iv) No specular reflection between surfaces

A typical situation is shown in cross section in Fig 6 , for the case where $\gamma < E$

and where γ lies in the particular range

$$5w/2(h^1 - h) \geq \cot\gamma > 3w/2(h^1 - h)$$

[we note that $\cot E = w/2\ (h^1 - h)$]

The direct radiation reaching the ground may now be broken down into a series of uniform strips parallel to the side of the house

The first strip XX^1 (located by points such that

$GX = h \cot\gamma$; $GX^1 = w + h \cot\gamma$) has passed through the single roof surface $(n = 1)$.

The second strip X^1X^{11} (located by the additional point such that

$GX^{11} = 2w + h \cot\gamma$) has passed through the three roof surfaces $(n = 1,2,3)$

The third strip $X^{11}X^{111}$ (located by the further point X^{111}, such that $GX^{111} = w/2 + h^1 \cot\gamma$) has passed through the five roof surfaces $(n = 1,2,3,4,5)$.

If for a given solar elevation (γ), the transmission coefficient for direct radiation through an odd numbered surface is indicated as (τ_1), and that through an even numbered surface as (τ_2), then the overall transmission coefficients appropriate to the strips are

$$(XX^{1}) \quad - \quad (\tau_1)$$

$$(X^{1}X^{11}) \quad - \quad (\tau_1)^2(\tau_2)$$

$$(X^{11}X^{111}) \quad - \quad (\tau_1)^3(\tau_2)^2$$

In the more general case of Fig.6 i.e., when $\theta \neq 0$ (shown in isometric form in Fig. 7) the problem is to obtain a general expression for the proportion of the individual strips that fall within the floor of the house. To assist in this, we need to locate the points X, X^{1}, X^{11}, X^{111}.

In Figs. 8 and 9, we show details of part of Fig.6, the case for $\theta = 0$

In the triangle STP of Fig 8(a) we have

$$\frac{TP}{\text{Sin}\,(90 - E)} = \frac{w \tan\gamma}{\text{Sin}\,(\gamma + E)}$$

$$TP = \frac{w \tan\gamma \cos E}{\text{Sin}\,(\gamma + E)}$$

$$\text{and } x = SQ = TP \cos\gamma$$

$$\boxed{x = \frac{w\,\text{Sin}\,\gamma \cos E}{\text{Sin}\,(\gamma + E)}}$$

$$\begin{aligned}\text{also} \quad H^{1} &= h + ST - TV \\ &= h + w \tan\gamma - TP \sin\gamma \\ &= h + w \tan\gamma - \frac{w \tan\gamma\,\text{Sin}\,\gamma \cos E}{\text{Sin}\,(\gamma + E)}\end{aligned}$$

$$\boxed{H^{1} = n + \frac{w\,\text{Sin}\,\gamma\,\text{Sin}\,E}{\text{Sin}\,(\gamma + E)}}$$

If we repeat the exercise for the distances (x^{11}), (H^{11}) of Fig. 6 we find

$$\boxed{x^{11} = \frac{2\,w \cdot \text{Sin}\,\gamma \cos E}{\text{Sin}\,(\gamma + E)}}$$

$$\boxed{H^{11} = h + \frac{2\,w\,\text{Sin}\,\gamma\,\text{Sin}\,E}{\text{Sin}\,(\gamma + E)}}$$

In the more general case, $\theta \neq 0$, a detail of which is shown in Fig. 10 we have

In triangle PQT, $QT = (H^1 - h)\cot\gamma$

$$= QR/\cos\theta \qquad \text{from triangle QRT.}$$

$$\text{or } (H^1 - h)\cot\gamma = (w - x^1)/\cos\theta$$

Again in triangle SPQ

$$(H^1 - h) = x^1 \tan E$$

Eliminating $(H - h)$ from these two expressions, we have

$$\frac{(w - x^1)}{\cos\theta\cot\gamma} = x^1 \tan E$$

$$(w - x^1) = x^1 \cot\gamma\tan E\cos\theta$$

$$w = x^1 (1 + \cot\gamma\tan E\cos\theta)$$

$$x^1 = w\left[\frac{1}{1 + \cot\gamma\tan E\cos\theta}\right]$$

and $H^1 = h + x^1 \tan E$

$$H^1 = h + \frac{w\tan E}{1 + \cot\gamma\tan E\cos\theta}$$

(which when $\theta = 0$, reduces to the value for (x^1), (H^1) found in the case of Figs 8 and 9)

It follows then that the coordinates in Fig. 7 of the points X, X^1, X^{11}, X^{111}, measured from the sides of the house are

$$X \equiv \left.\begin{array}{l} h\cot\gamma\cos\theta \\ h\cot\gamma\sin\theta \end{array}\right\}$$

$$X^1 \equiv \left.\begin{array}{l} x^1 + H^1\cot\gamma\cos\theta \\ \quad\; H^1\cot\gamma\sin\theta \end{array}\right\}$$

$$X^1 \equiv \frac{w}{1 + \cot\gamma\tan E\cos\theta} + \left[h + \frac{w\tan E}{1 + \cot\gamma\tan E\cos\theta}\right]\cot\gamma\cos\theta$$

$$\text{and} \qquad \left[h + \frac{w\tan E}{1 + \cot\gamma\tan E\cos\theta}\right]\cot\gamma\sin\theta$$

$$X^1 \equiv \left.\begin{array}{l} w + h\cot\gamma\cos\theta \\ \text{and } \left[\dfrac{w\tan E\cot\gamma\sin\theta}{1 + \cot\gamma\tan E\cos\theta}\right] + h\cot\gamma\sin\theta \end{array}\right\}$$

$$X^{11} = 2\,w + h \cot\gamma \cos\theta \quad \Big)$$

$$\text{and} \left[\frac{2\,w \tan E \cot\gamma \operatorname{Sin}\theta}{1 + \cot\gamma \tan E \operatorname{Cos}\theta}\right] + h \cot\gamma \operatorname{Sin}\theta \quad \Big)$$

$$X^{111} = w/2 + h_1^1 \cot\gamma \cos\theta)$$

$$h^1 \cot\gamma \operatorname{Sin}\theta)$$

Considering an individual strip such as $XX^1 Z^1Z$, then Fig. 11 shows in in plan, alternative configurations of floor area and radiation pattern on the ground, that we shall need to consider (There is of course the additional situation where $XX^1 Z^1Z$ lies completely outside the floor of the house).

In Fig. 11, Case I the proportion of the strip intercepted by the floor of the house is

$$(F) = 1 - \frac{XY + X^1Y^1}{2\,l}$$

$$= 1 - \frac{h \cot\gamma \operatorname{Sin}\theta}{2\,l} \left[h + \frac{w \tan E}{1 + \cot\gamma \tan E \operatorname{Cos}\theta}\right] \frac{\cot\gamma \operatorname{Sin}\theta}{2\,l}$$

$$(F) = 1 - \frac{h \cot\gamma \sin\theta}{l} - \frac{(w/2) \cot\gamma \tan E \operatorname{Sin}\theta}{l\,(1 + \cot\gamma \tan E \operatorname{Cos}\theta)}$$

The conditions for Fig.11, Case I, are evidently that X, X^1 should lie within the house

ie that $(h \cot\gamma \cos\theta) \leq Nw/2$

and $(h \cot\gamma \operatorname{Sin}\theta) < l$

also that $(w + h \cot\gamma \cos\theta) \leq Nw/2$

and $\left(\frac{w \tan E \cot\gamma \operatorname{Sin}\theta}{(1 + \cot\gamma \tan E \operatorname{Cos}\theta} + h \cot\gamma \operatorname{Sin}\theta\right) < l$

For the remaining cases of Fig. 11, the evaluation of (F) is evidently more complex (a treatment was given in the earlier memorandum). However, for the multi-span house, the values of (γ), and so of I_γ, which will lead to the Cases II and III in practice, must be small. Since in these cases, $(F) \ll 1$, we make the assumption that for cases other than Case I, we may neglect the contribution to the radiation received within the floor of the house.

For cases other than Case I, we write (F) = o and the conditions under which this applies are evidently

X^{1} outside the house i.e. $(w + h \cot\gamma \cos\theta) > Nw/2$

$$\text{or} \quad \left(\frac{w \tan E \cot\gamma \operatorname{Sin}\theta}{1 + \cot\gamma \tan E \cos\theta} + h \cot\gamma \operatorname{Sin}\theta \right) \geq 1$$

Applying similar considerations to the strip $X^{1}\ X^{11}\ Z^{11}\ Z^{1}$ we take (F) = o when

X^{11} lies outside the house i.e. when

$$(2w + h \cot\gamma \cos\theta) > Nw/2$$

$$\text{or} \quad \left(\left[\frac{2w \tan E \cot\gamma \operatorname{Sin}\theta}{1 + \cot\gamma \tan E \cos\theta} \right] + h \cot\gamma \operatorname{Sin}\theta \right) \geq 1.$$

When these conditions are not met for the strip $X^{1}\ X^{11}\ Z^{11} Z^{1}$ (F) for this strip is given by

$$(F) = 1 - \frac{X^{1}Y^{1} + X^{11}Y^{11}}{21}$$

$$(F) = 1 - \frac{h \cot\gamma \operatorname{Sin}\theta}{1} - \frac{(3w/2) \cot\gamma \tan E \operatorname{Sin}\theta}{1\ (1 + \cot\gamma \tan E \cos\theta)}$$

Repeating the arguments for the strip $X^{11}X^{111}\ Z^{111}Z^{11}$, we take

(F) = o when X^{111} lies ontside the house

i.e. when

$$(w/2 + h^{1} \cot\gamma \cos\theta) > Nw/2$$

$$\text{or} \quad (h^{1} \cot\gamma \operatorname{Sin}\theta) \geq 1.$$

When these conditions are not met for the strip $X^{11}X^{111}\ Z^{111}\ Z^{11}$, (F) for this strip is given by

$$(F) = 1 - \frac{X^{11}Y^{11} + X^{111}\ Y^{111}}{2}$$

$$(F) = 1 - \frac{(h + h^{1}) \cot\gamma \operatorname{Sin}\theta}{21} - \frac{w \cot\gamma \tan E \operatorname{Sin}\theta}{1\ (1 + \cot\gamma \tan E \cos\theta)}$$

Combining the results so far for the case where

(i) $n = 1$

(ii) $90^{\circ} \geq \theta \geq 0^{\circ}$; $(A_{p1}) = o$

(iii) $5w/2\ (h^{1} - h) \geq \cot\gamma > 3w/2\ (h^{1} - h)$

(iv) the point X^{111} lies within the house,

Then the quantity (τF) in our general expression takes the form

$$(\tau F) = (\tau_1) \left\{ 1 - \frac{h \cot\gamma \operatorname{Sin}\theta}{1} - \frac{(w/2) \cot\gamma \tan E \operatorname{Sin}\theta}{1\ (1 + \cot\gamma \tan E \cos\theta)} \right)$$

$$+ (T_1)^2 (T_2) \left(1 - \frac{h \cot\gamma \sin\theta}{l} - \frac{(3\,w/2)\cot\gamma \tan E \,\mathrm{Sin}\,\theta}{l(1 + \cot\gamma \tan E \cos\theta)} \right)$$

$$+ (T_1)^3 (T_2)^2 \left(1 - \frac{(h + h^1)\cot\gamma \,\mathrm{Sin}\,\theta}{2l} - \frac{w \cot\gamma \tan E \,\mathrm{Sin}\,\theta}{l\,(1 + \cot\gamma \tan E \cos\theta)} \right)$$

One of our problems is to generalise this relationship to include

all roof surfaces

all values of θ

all values of γ

Our general expression for (β), the angle of incidence of radiation onto any roof surface (cf. Ag. Mem. 277) will show in the present example, as drawn in Fig. 12. that

at roof surface n = 1 , $\cos\beta_1 > 0$; $(A_{p1}) = 0$

at roof surface n = 2 , $\cos\beta_2 < 0$; $A_{p1} = -180^c$

A negative value for $\cos\beta_2$ implies simply that the direct solar beam ~~imp~~ impinges on the underside of the surface n = 2. Obviously we must take the modulus $|\cos\beta_2|$ to evaluate (T_2)

The range of values of (T_1) and (T_2) is 0 to 1, with a median value of ~~0~~ 0.5. As $\cos\beta_1$, increases, $\cos\beta_2$ decreases (and vice versa). A comparison of orders of magnitude suggest we might neglect terms involving the products $(T_1)^4 (T_2)^3$, $(T_1)^3 (T_2)^4$, or higher powers.

9.1. Direct Radiation – the general evaluation of (τF)

9.1.1. Case Roof surface $n=1$; $\theta = t-(A\phi_1)$; $90^\circ \geq \theta \geq 0^\circ$

OR Roof surface $n=N$; $\theta = t-(A\phi_N)$; $90^\circ \geq \theta \geq 0^\circ$

ie the end surface n=1 or n=N faces the sun.

(we note $(A\phi_n) = (A\phi_1) - 180^\circ$)

9.1.1.(a) $\geq$

$$\boxed{w/2(h'-h) \geq \cot\gamma}$$

The situation is shown in cross-section, in Fig 13(a), for the case when $\theta = 0$. In plan, the situation for the more general case $(\theta \neq 0)$ is that of Fig 11.

When X' lies within the house

ie when

$$\boxed{(w/2 + h' \cot\gamma) \leq Nw/2}$$

and

$$\boxed{h' \cot\gamma \sin\theta\ ) < l}$$

then $(\tau) = (\tau_1)$

$$(F) = 1 - \frac{XY + X'Y'}{2l}$$

$$= 1 - \frac{(h+h')\cot\gamma \sin\theta}{2l}$$

$$(\tau F) = (\tau_1)\left[1 - \frac{(h+h')\cot\gamma \sin\theta}{2l}\right] \quad \text{when } (n = 1)$$

$$(\tau F) = (\tau_2)\left[1 - \frac{(h+h')\cot\gamma \sin\theta}{2l}\right] \quad \text{when } (n = N)$$

When the above conditions for X' are not met

$$(\tau F) = 0$$

9.1.1.(b)

$$\boxed{3w/2(h'-h) \geq \cot\gamma > w/2(h'-h)}$$

The situation is shown in cross section in Fig 13(b) for the case when $\theta = 0$. In plan for the more general case $(\theta \neq 0)$, each of the strips XX'Y'Y, X'X"Y"Y', will fall within one of the categories of Fig 11

X" falls within the house (n = 1)

The conditions for this are that

$$\boxed{(w/2 + h' \cot\gamma \cos\theta) \leq Nw/2}$$

and

$$\boxed{(h' \cot\gamma \sin\theta) < 1}$$

When these conditions are met,

$$(\tau F) = (\tau_1) \left[\frac{1 - XY + X'Y'}{21} \right] + (\tau_1)^2 (\tau_2) \left[\frac{1 - X'Y' + X''Y''}{21} \right]$$

$$(\tau F) = (\tau_1) \left[1 - \frac{h \cot\gamma \sin\theta}{1} - \frac{(w/2) \cot\gamma \tan\epsilon \sin\theta}{1(1 + \cot\gamma \tan\epsilon \cos\theta)} \right]$$

$$+ (\tau_1)^2 (\tau_2) \left[1 - \frac{(w/2) \cot\gamma \tan\epsilon \sin\theta}{1(1 + \cot\gamma \tan\epsilon \cos\theta)} - \frac{(h + h') \cot\gamma \sin\theta}{21} \right]$$

X" falls outside the house (n=1)

X' falls within the house

The conditions for this are

$$\boxed{(w/2 + h' \cot\gamma \cos\theta) > Nw/2}$$

OR

$$\boxed{h' \cot\gamma \sin\theta) \geq 1}$$

together with

$$\boxed{w + h \cot\gamma \cos\theta) \leq Nw/2}$$

and

$$\boxed{\left(\frac{w \cot\gamma \tan\epsilon \sin\theta}{1 + \cot\gamma \tan\epsilon \cos\theta} + h \cot\gamma \sin\theta \right) < 1}$$

Now
by our convention of section 9

$$(\tau F) = (\tau_1) \left[\frac{1 - XY + X'Y'}{21} \right] \quad \text{only}$$

and $$(\tau F) = (\tau_1) \left[1 - \frac{h \cot\gamma \sin\theta}{1} - \frac{(w/2) \cot\gamma \tan\epsilon \sin\theta}{1(1 + \cot\gamma \tan\epsilon \cos\theta)} \right]$$

X' falls outside the house (n = 1)

The conditions for this are that

$$\boxed{(w + h \cot\gamma \cos\theta) > Nw/2}$$

OR

$$\boxed{\left(\frac{w \cot\gamma \tan\epsilon \sin\theta}{1 + \cot\gamma \tan\epsilon \cos\theta} + h \cot\gamma \sin\theta\right) \geq 1}$$

Now $(\tau F) = 0$

For $(n=N)$, (τ_1) and (τ_2) in the above expressions for (τF) are interchanged

9.1.1.(c) ______

$$\boxed{5w/2(h'-h) \geq \cot\gamma > 3w/2\ (h'-h)}$$

The situation is shown in cross section in Fig 13(c) for the case when $\theta = 0$. In plan for the more general case $(\theta \neq 0)$, each of the strips XX'Y'Y, X'X"Y"Y', X"X"'Y"'Y" of Fig 10 will fall within one of the categories of Fig 11.

X"' falls within the house (n = 1)

The conditions for this are that

$$\boxed{(w/2 + h' \cot\gamma \cos\theta) \leq Nw/2}$$

and

$$\boxed{(h' \cot\gamma \sin\theta) < 1}$$

When these conditions are met,

$$(\tau F) = (\tau_1)\left[1 - \frac{XY + X'Y'}{21}\right]$$

$$+ (\tau_1)^2(\tau_2)\left[1 - \frac{X'Y' + X''Y''}{21}\right]$$

$$+ (\tau_1)^3(\tau_2)^2\left[1 - \frac{X''Y'' + X'''Y'''}{21}\right]$$

$$(\tau_F) = (\tau_1)\left[1 - \frac{h\cot\gamma\sin\theta}{l} - \frac{(w/2)\cot\gamma\tan\epsilon\sin\theta}{l\,(1+\cot\gamma\tan\epsilon\cos\theta)}\right]$$

$$\cancel{(\quad)} + (\tau_1^2)(\tau_2)\left[1 - \frac{h\cot\gamma\sin\theta}{l} - \frac{(3w/2)\cot\gamma\tan\epsilon\sin\theta}{l\,(1+\cot\gamma\tan\epsilon\cos\theta)}\right]$$

$$\cancel{(\quad)} + (\tau_1)^3(\tau_2)^2\left[1 - \frac{(h+h')\cot\gamma\sin\theta}{2l} - \frac{w\cot\gamma\tan\epsilon\sin\theta}{l\,(1+\cot\gamma\tan\epsilon\cos\theta)}\right]$$

X''' falls outside the house $(n = 1)$

X'' falls within the house

The conditions for this are

$$\boxed{(w/2 + h'\cot\gamma\cos\theta) > NW/2}$$

OR

$$\boxed{(h'\cot\gamma\sin\theta) \geqslant l}$$

together with

$$\boxed{(2w + h\cot\gamma\cos\theta) \leqslant NW/2}$$

and

$$\boxed{\left(\frac{2w\cot\gamma\tan\epsilon\sin\theta}{1+\cot\gamma\tan\epsilon\cos\theta} + h\cot\gamma\sin\theta\right) < l}$$

when these conditions are met

$$(\tau_F) = (\tau_1)\left[1 - \frac{h\cot\gamma\sin\theta}{l} - \frac{(w/2)\cot\gamma\tan\epsilon\sin\theta}{l\,(1+\cot\gamma\tan\epsilon\cos\theta)}\right]$$

$$\cancel{(\quad)} + (\tau_1)^2(\tau_2)\left[1 - \frac{h\cot\gamma\sin\theta}{l} - \frac{(3w/2)\cot\gamma\tan\epsilon\sin\theta}{l\,(1+\cot\gamma\tan\epsilon\cos\theta)}\right]$$

X'' falls outside the house $(n = 1)$
X' falls within the house

The conditions for this are

$$\boxed{(2w + h \cot \gamma \cos \theta) > Nw/2}$$

OR

$$\boxed{\left(\frac{2w \cot \gamma \tan \epsilon \sin \theta}{1 + \cot \gamma \tan \epsilon \cos \theta} + h \cot \gamma \sin \theta\right) \geq l}$$

together with

$$\boxed{(w + h \cot \gamma \cos \theta) \leq Nw/2}$$

and

$$\boxed{\left(\frac{w \cot \gamma \tan \epsilon \sin \theta}{1 + \cot \gamma \tan \epsilon \cos \theta} + h \cot \gamma \sin \theta\right) < l}$$

When these conditions are met.

$$(\tau_F) = (\tau_1)\left[1 - \frac{h \cot \gamma \sin \theta}{l} - \frac{(w/2) \cot \gamma \tan \epsilon \sin \theta}{l\,(1 + \cot \gamma \tan \epsilon \cos \theta)}\right]$$

X' falls outside the house (n = 1)

The conditions for this are

$$\boxed{(w + h \cot \gamma \cos \theta) > Nw/2}$$

OR

$$\boxed{\left(\frac{w \cot \gamma \tan \epsilon \sin \theta}{1 + \cot \gamma \tan \epsilon \cos \theta} + h \cot \gamma \sin \theta\right) \geq l}$$

and when these conditions are met,

$$(\tau_F) = 0$$

For the case $(n = N)$, (Υ_1) and (Υ_2) in the above expressions for (Υ_F) are interchanged.

6.1.1. (d)

$$\boxed{7w/2(h'-h) \geqslant \cot\gamma \; 5w/2 \quad (h'-h)}$$

In cross-section for the case when $\theta=0$, the situation posed is accommodated by the introduction of a further point X'''' in Fig 13 (c).

Assuming in the general case $(\theta \neq 0)$, that X'''' still lies within the floor of the house we have, as before.

$$(\Upsilon_f) = (\Upsilon_1)\left[1 - \frac{XY + X'Y'}{2l}\right]$$
$$+ (\Upsilon_1)^2 . (\Upsilon_2)\left[1 - \frac{X'Y' + X''Y''}{2l}\right]$$
$$+ (\Upsilon_1)^3 . (\Upsilon_2)^2\left[1 - \frac{X''Y'' + X'''Y'''}{2l}\right]$$

+ higher powers of $(\Upsilon_1) . (\Upsilon_2)$ which are neglected.

X'''' falls within the floor of the house $(n = 1)$

The conditions for this are that

$$\boxed{(w/2 + h' \cot\gamma \cos\theta) \leqslant Nw/2)}$$

$$\boxed{(h' \cot\gamma \sin\theta) < l}$$

when these conditions are met.

$$(\Upsilon F) = (\Upsilon_1)\left[1 - \frac{h \cot\gamma \sin\theta}{l} - \frac{(w/2) \cot\gamma \tan\epsilon \sin\theta}{l(1 + \cot\gamma \tan\epsilon \cos\theta)}\right]$$
$$+ (\Upsilon_1)^2 . (\Upsilon_2)\left[1 - \frac{h \cot\gamma \sin\theta}{l} - \frac{(3w/2) \cot\gamma \tan\epsilon \sin\theta}{l(1+ \cot\gamma \tan\epsilon \cos\theta)}\right]$$
$$+ (\Upsilon_1)^3 . (\Upsilon_2)^2\left[1 \quad \frac{h \cot\gamma \sin\theta}{l} - \frac{(5w/2) \cot\gamma \tan\epsilon \sin\theta}{l(1+ \cot\gamma \tan\epsilon \cos\theta)}\right]$$

X'''' falls outside the house (n = 1)
X''' falls within the house

The conditions for this are

$$\boxed{(w/2 + h' \cot \gamma \cos \theta) > Nw/2}$$

OR

$$\boxed{(h' \cot \gamma \sin \theta) \geq l}$$

together with

$$\boxed{(3w + h \cot \gamma \cos \theta) \leq Nw/2}$$

and

$$\boxed{\left(\frac{3w \cot \gamma \tan \varepsilon \sin \theta}{1 + \cot \gamma \tan \varepsilon \cos \theta} + h \cot \gamma \sin \theta\right) < l}$$

when these conditions are met, again

$$(\Upsilon F) \sim (\Upsilon_1) \left[1 - \frac{h \cot \gamma \sin \theta}{l} - \frac{(w/2) \cot \gamma \tan \varepsilon \sin \theta}{l\,(1 + \cot \gamma \tan \varepsilon \cos \theta)}\right]$$

$$-$$

$$(\Upsilon_1)^2 . (\Upsilon_2) \left[1 - \frac{h \cot \gamma \sin \theta}{l} - \frac{(3w/2) \cot \gamma \tan \varepsilon \sin \theta}{l\,(1 + \cot \gamma \tan \varepsilon \cos \theta)}\right]$$

$$(\Upsilon_1)^3 . (\Upsilon_2)^2 \left[1 - \frac{h \cot \gamma \sin \theta}{l} - \frac{(5w/2) \cot \gamma \tan \varepsilon \sin \theta}{l\,(1 + \cot \gamma \tan \varepsilon \cos \theta)}\right]$$

X'''' falls outside the house (n = 1)
X'' falls within the house

The conditions for this are

$$\boxed{(3w + h \cot \gamma \cos \theta) > Nw/2}$$

OR

$$\boxed{\left(\frac{3w \cot \gamma \tan \varepsilon \sin \theta}{1 + \cot \gamma \tan \varepsilon \cos \theta} + h \cot \gamma \sin \theta\right) \geq l}$$

together with

$$\boxed{(2w + h \cot \gamma \cos \theta) \leq Nw/2}$$

$$\boxed{\left(\frac{2w \cot \gamma \tan \varepsilon \sin \theta}{1 + \cot \gamma \tan \varepsilon \cos \theta} + h \cot \gamma \sin \theta \right) < l}$$

$$\text{Now } (\tau F) = (\tau_1) \left[1 - \frac{h \cot \gamma \sin \theta}{l} - \frac{(w/2) \cot \gamma \tan \varepsilon \sin \theta}{l\,(1 + \cot \gamma \tan \varepsilon \cos \theta)} \right]$$

$$+ (\tau_1)^2 \cdot (\tau_2) \left[1 - \frac{h \cot \gamma \sin \theta}{l} - \frac{(3w/2) \cot \gamma \tan \varepsilon \sin \theta}{l\,(1 + \cot \gamma \tan \varepsilon \cos \theta)} \right]$$

X'' falls outside the house (n = 1)

X' falls within the house

The conditions for this are

$$\boxed{(2w + h \cot \gamma \cos \theta) > N w/2}$$

$$\boxed{\frac{(2w \cot \gamma \tan \varepsilon \sin \theta}{1 + \cot \gamma \tan \varepsilon \cos \theta} + h \cot \gamma \sin \theta \geq l}$$

together with

$$\boxed{(w + h \cot \gamma \cos \theta \leq Nw/2}$$

and

$$\boxed{\left(\frac{w \cot \gamma \tan \varepsilon \sin \theta}{1 + \cot \gamma \tan \varepsilon \cos \theta} + h \cot \gamma \sin \theta \right) < l}$$

$$(\tau F) = (\tau_1) \left[1 - \frac{h \cot \gamma \sin \theta}{l} - \frac{(w/2) \cot \gamma \tan \varepsilon \sin \theta}{l\,(1 + \cot \gamma \tan \varepsilon \cos \theta)} \right]$$

X' falls outside the house (n = 1)

The conditions for this are

$$\boxed{(w + h \cot \gamma \cos \theta) > Nw/2}$$

OR

$$\boxed{\left(\frac{w \cot \gamma \tan \varepsilon \sin \theta}{1 + \cot \gamma \tan \varepsilon \cos \theta} + h \cot \gamma \sin \theta \right) \geq l}$$

Now $(\tau F) = 0$

For the case $(n = N)$, (τ_1) and (τ_2) in the above expressions for (τF) are interchanged.

9.1.1.(e) $$\boxed{9w/2(h'-h) \geq \cot\gamma > 7w/2(h'-h)}$$

Neglect of powers higher than $(\gamma_1)^3 \; (\gamma_2)^2$ evidently leads to the same expressions for (γF) as were developed for case (d). The same is true for still smaller values of (γ) [such that $\cot\gamma > 9 \; w/2(h'-h)$].

So we have, provided that X''' falls within the floor of the house i.e. provided

$$\boxed{(3w + h \cot\gamma \cos\theta \leq Nw/2}$$

and $$\boxed{\left(\frac{3w \cot\gamma \tan\varepsilon \sin\theta}{1 + \cot\gamma \tan\varepsilon \cos\theta} + h \cot\gamma \sin\theta\right) < 1}$$

$$(\gamma F) = (\gamma_1)\left[1 - \frac{h \cot\gamma \sin\theta}{1} - \frac{(w/2) \cot\gamma \tan\varepsilon \sin\theta}{1 \; (1 + \cot\gamma \tan\varepsilon \cos\theta)}\right]$$

$$+ (\gamma_1)^2(\gamma_2)\left[1 - \frac{h \cot\gamma \sin\theta}{\ell} - \frac{(3w/2) \cot\gamma \tan\varepsilon \sin\theta}{1 \; (1 + \cot\gamma \tan\varepsilon \cos\theta)}\right]$$

$$+ (\gamma_1)^3(\gamma_2)^2\left[1 - \frac{h \cot\gamma \sin\theta}{\ell} - \frac{(5w/2) \cot\gamma \tan\varepsilon \sin\theta}{1 \; (1 + \cot\gamma \tan\varepsilon \cos\theta)}\right] \quad (n = 1)$$

If X''' falls outside the house (n = 1)

x'' falls within the house

The conditions for this are

$$\boxed{(3w + h \cot\gamma \cos\theta) > Nw/2}$$

OR

$$\boxed{\left(\frac{3w \cot\gamma \tan\varepsilon \sin\theta}{1 + \cot\gamma \tan\varepsilon \cos\theta} + h \cot\gamma \sin\theta\right) \geq \ell}$$

together with

$$\boxed{(2w + h \cot\gamma \cos\theta \leq Nw/2}$$

$$\boxed{\frac{(2w \cot\gamma \tan\varepsilon \sin\theta}{(1 + \cot\gamma \tan\varepsilon \cos\theta} + h \cot\gamma \sin\theta < \ell}$$

then $$(\Upsilon F) = (\Upsilon_1)\left[1 - \frac{h\cot\gamma\ \sin\theta}{l} - \frac{(w/2)\cot\gamma\ \tan\varepsilon\sin\theta}{l\,(1+\cot\gamma\ \tan\varepsilon\cos\theta)}\right]$$

$$+ (\Upsilon_1)^2(\Upsilon_2)\left[1 - \frac{h\cot\gamma\ \sin\theta}{l} - \frac{(3w/2)\cot\gamma\ \tan\varepsilon\sin\theta}{l\,(1+\cot\gamma\ \tan\varepsilon\cos\theta)}\right]$$

If X'' falls outside the house (n = 1)

X' falls within the house

The conditions for this are

$$\boxed{(2w + h\cot\gamma\ \cos\theta) > Nw/2)}$$

OR

$$\boxed{\left(\frac{2w\cot\gamma\ \tan\varepsilon\ \sin\theta}{1+\cot\gamma\ \tan\varepsilon\cos\theta} + h\cot\gamma\ \sin\theta\right) \geq l}$$

together with

$$\boxed{(w + h\cot\gamma\ \cos\theta) \leq Nw/2}$$

$$\boxed{\left(\frac{w\cot\gamma\tan\varepsilon\sin\theta}{1+\cot\gamma\ \tan\varepsilon\cos\theta} + h\cot\gamma\ \sin\theta\right) < l}$$

then $$(\Upsilon F) = (\Upsilon_1)\left[1 - \frac{h\cot\gamma\ \sin\theta}{l} - \frac{(w/2)\cot\gamma\ \tan\varepsilon\sin\theta}{l\,(1+\cot\gamma\ \tan\varepsilon\cos\theta)}\right]$$

If X' falls outside the house (n = 1)

If either

$$\boxed{(w + h\cot\gamma\ \cos\theta) > Nw/2}$$

OR

$$\boxed{\left(\frac{w\cot\gamma\ \tan\varepsilon\sin\theta}{1+\cot\gamma\ \tan\varepsilon\cos\theta} + h\cot\gamma\ \sin\theta\right) \geq l}$$

then $(\Upsilon F) = 0$

For the case (n=N), (Υ_1) and (Υ_2) in the above expressions for (ΥF) must be interchanged.

9.1.2. Case Roof surface $n = 1$; $\theta = t - (Ap_1)$; $\theta > 90^\circ$

OR Roof surface $n = N$; $\theta = t - (A p_N)$; $\theta > 90^\circ$

ie the end surface $n = 1$ or $n = N$ faces away from the sun.

The only situation that we need consider in any detail is that shown in cross-section in Fig 14, for the case $\theta = 0$ and in plan for the more general case ($\theta \neq 0$) in Fig. 11 (Case II). Case I of Fig 11 will not arise in practice in the British Isles and for Case III, we continue the convention (ΥF) = 0.

For the present we continue the convention that for the situation under discussion (Fig 14), we may take (ΥF) = 0.

However, since for the smaller multi-span house it may be desirable to consider orders of magntitude a little more closely, we present the analysis that would enable us to compute (ΥF) if this were thought necessary.

Consider the case ($n = N$)

We assume now that the point X' will always lie outside the house.

If X' is such that

$$\boxed{(h \cot \gamma \sin\theta) \geq 1}$$

then we deal with Fig 11. Case III and (ΥF) = 0

If X' is such that

$$\boxed{(h \cot \gamma \sin \theta) < 1}$$

and provided that X lies within the house
ie that

$$\boxed{(h' \cot \gamma \cos \theta) < w/2}$$

and

$$\boxed{(h' \cot \gamma \sin \theta) < 1}$$

then these conditions specify Fig 11, Case II

/For either $\boxed{(h' \cot\gamma \cos\theta) \geq w/2}$

OR

$\boxed{(h' \cot\gamma \sin\theta \geq l}$

then X lies outside the house and $(\tau F) = 0$

In the general case (Fig 11, Case II) the proportion of the strip XX' intercepted by the floor of the house is

$$(F) = \frac{\text{Area XUFW}}{\text{Area XX' ZZ'}}$$

$$= \frac{\text{Area XX' Z'Z} - \text{Area U'X'Z' U'} \quad \text{Area WW' Z'Z} + \text{Area FW'Z'U'}}{\text{Area XX' ZZ'}}$$

Now (as in Ag Mem 277)

Area XX' Z'Z $= l\left[(w/2) - (h'-h)\cot\gamma\cos\theta\right]$
Area UX'Z'U' $= 1\, h \cot\gamma \cos\theta$
Area WW'Z'Z $= \frac{1}{2}\left[(w/2) - (h'-h)\cot\gamma\cos\theta\right]\left[(h' + h)\cot\gamma\sin\theta\right]$
Area FW'Z'U' $\frac{1}{2}\left[h\cot\gamma\cos\theta\right]\left[(h'+h)\cot\gamma\sin\theta\right]$

From which $(F) = \left[1 - \left|\frac{(h'-h)\cot\gamma\sin\theta}{21}\right|\right]\left[1 - \left|\frac{h\cot\gamma\cos\theta}{(w/2) - |(h'-h)\cot\gamma\cos\theta|}\right|\right]$

when $(n = 1)$ we take $(\tau) = (\tau_1)$
$(n = N)$ we take $(\tau) = (\tau_2)$

9.1.3 Case Roof Surface $n = n$; $0 = t - (A_{p1})$; $90^\circ \geq \theta \geq 0^\circ$

ie the $(n)^{th}$ roof surface when the upsun surface is $n = 1$.

/The cases $n = 1$ or $n = N$ already discussed are excluded/

Up until this juncture we have been concerned with the overall area of the surface that is being examined for transmission into the house. We now must take account of the possibility of shading of roof surfaces by those which are up-sun, and the quantity (a) in our basic expression having to be replaced by another.

9.1.3 (a) $\boxed{\gamma \geq E}$ ie $\boxed{w/2(h^1 - h) \geq \cot \gamma}$

(transmitting area is full roof surface)

$\boxed{n \text{ is odd}}$ ie faces the sun

For the case when $\theta = 0$, the situation is shown in cross section in Fig. 15 (a). In plan for the more general case of $(\theta \neq 0)$, the situation is that of Fig. 11 Case I

If X^1 lies outside the house, the conditions for this are

$$h^1 \cot \gamma \cos \theta + nw/2 > Nw/2$$

ie $$\boxed{h^1 \cot \gamma \cos \theta > (N - n)\, w/2}$$

or $$\boxed{h^1 \cot \gamma \sin \theta \geq 1}$$

and by our convention, $(\tau F) = 0$

If X^1 lies within the house, then the conditions for this are that

$$\boxed{h^1 \cot \gamma \cos \theta \leq (N-n)\, w/2}$$

$$\boxed{h^1 \cot \gamma \sin \theta < 1}$$

$(\tau) = (\tau_1)$ and (F) follows from the analysis given for the similar case when $(n = 1)$, so that

$$(\tau F) = (\tau_1) \left[1 - \frac{(h + h^1) \cot \gamma \sin \theta}{21} \right]$$

$\boxed{\text{n is even}}$ ie faces away from the sun

As in the case when n is odd, we refer to Fig 15 (a) and Fig 11, Case I. The point X^1 lies outside the house if

$$nw/2 + h \cot \gamma \cos \theta > Nw/2$$

ie $$\boxed{h \cot \gamma \cos \theta > (N - n) w/2}$$

or $$\boxed{h \cot \gamma \sin \theta \geq 1}$$

If either of these conditions are met, we take $(\tau F) = 0$

If X^1 lies within the house,

$$\boxed{h \cot \gamma \cos \theta \leq (N - n) w/2}$$

and $$\boxed{h \cot \gamma \sin \theta < 1}$$

$(\tau) = (\tau_2)$ and (F) follows from the analysis given for the similar case when $(n = N)$, so that

$$(\tau F) = (\tau_2) \left[1 - \frac{(h + h^1) \cot \gamma \sin \theta}{21} \right]$$

9.1.3 (b)

$\boxed{\gamma < E}$ $\boxed{3w/2(h^1 - h) \geq \cot \gamma > w/2 (h^1 - h)}$

$\boxed{\text{n is odd}}$ ie faces the sun

The general situation is shown in Fig 16. The detail shown in Fig 9 also applies, so that, as before in section 9, the height of the point P is given by

$$H^1 = h + \frac{w \tan E}{1 + \cot\gamma \tan E \cos\theta}$$

and the horizontal distance corresponding to SP is

$$x^1 = \frac{w}{1 + \cot\gamma \tan E \cos\theta}$$

In Fig 17, the appropriate part of Fig 16 is shown in plan.

We see that

$$X^{11} Z^{11} = A_n A_n^{\ 1} = w \tan\theta$$

$$X^1 Z^1 \simeq PP^1 = (w/2 + x^1) \tan\theta$$

$$X^1 Z^1 = \left[(w/2) + \frac{w}{1 + \cot\gamma \tan E \cos\theta} \right] \tan\theta$$

$$SA_n = (w/2) \sec\theta$$

and the transmitting area $(\acute{a})$ of the surface SPA_n (the nth surface) for directly incident direct radiation is

$$(\acute{a}) = (w/2) \sec E \left[\frac{w \tan\theta + (w/2) \tan\theta}{2} \right]$$

$$(\acute{a}) = (3/8)\ w^2 \sec E \tan\theta$$

So that,

when $\tan \theta \leq 1/w$

$$(a)^1 = (3/8)\ w^2 \sec^2 E \tan \theta$$

Similarly,

when $1/(w/2) > \tan \theta > 1/w$

$$(a^1) = w\ 1 \sec E - (1^2/2) \cot \theta \sec E - (w^2/8) \tan \theta \sec E$$

Also,

when $\tan \theta \geqslant 1/(w/2)$

$$(a^1) = (a) = 1(w/2) \sec E$$

The locations of the points X, X^1, X^{11}, Z, Z^1, Z^{11} from the sides GH, GE of the house are

$$X \underline{=} \begin{array}{l} h \cot \gamma \cos \theta + (n - 1)(w/2) \\ h \cot \gamma \sin \theta \end{array}$$

$$X^1 \underline{=} \begin{array}{l} \left[h + \dfrac{w \tan E}{1 + \cot \gamma \tan E \cos \theta} \right] \cot \gamma \cos \theta + (n - 1)(w/2) \\ \left[h + \dfrac{w \tan E}{1 + \cot \gamma \tan E \cos \theta} \right] \cot \gamma \sin \theta \end{array}$$

$$X^{11} \underline{=} \begin{array}{l} h^1 \cot \gamma \cos \theta + n(w/2) \\ h^1 \cot \gamma \sin \theta \end{array}$$

$$Z \underline{=} \begin{array}{l} h \cot \gamma \cos \theta + (n - 1)\ w/2 \\ h \cot \gamma \sin \theta + (w/2) \tan \theta \end{array}$$

$$Z^1 \equiv \left[h + \frac{w \tan \varepsilon}{1 + \cot\gamma \tan \varepsilon \cos\theta} \right] \cot\gamma \cos\theta + (n - 1)(w/2)$$

$$\left[h + \frac{w \tan E}{1 + \cot\gamma \tan E \cos\theta} \right] \cot\gamma \sin\theta + \left[(w/2) + \frac{w}{1 + \cot\gamma \tan E \cos\theta} \right] \tan\theta$$

$$Z^{11} \equiv h^1 \cot\gamma \cos\theta + n\,(w/2)$$

$$\qquad h^1 \cot\gamma \sin\theta + w \tan\theta$$

We consider first the situation shown in plan in Fig 18, Case I (a), in which all parts of the areas $X X^1 Z^1 Z$, $X^1 X^{11} Z^{11} Z^1$ lie within the floor of the house. The conditions for this may be stated simply as those which require X^{11} and Z^{11} to lie within the floor of the house.

ie $$h^1 \cot\gamma \cos\theta + n(w/2) \leq N(w/2)$$

$$\boxed{h^1 \cot\gamma \cos\theta \leq (N - n)(w/2)}$$

and $$\boxed{h^1 \cot\gamma \sin\theta + w \tan\theta \leq 1}$$

Under these conditions the angle factor (F) = 1

In the situation of Fig. 18 (Case IIa) for the area $X^1 X^{11} Z^{11} Z$ ie in which X^{11} lies within the house, Z^{11} lies outside the house (and in which the area $X X^1 Z^1 Z$ still lies completely within the house), such a situation is specified by

$$\boxed{h^1 \cot\gamma \cos\theta \leq (N - n)(w/2)}$$

$$\boxed{h^1 \cot\gamma \sin\theta + w \tan\theta > 1}$$

together with

$$\boxed{h \cot\gamma \cos\theta + \frac{w \cot\gamma \tan E \cos\theta}{1 + \cot\gamma \tan E \cos\theta} + (n - 1)\, w/2 \leq N(w/2)}$$

$$\boxed{\left(h \cot\gamma \sin\theta + \frac{w \cot\gamma \tan E \sin\theta}{1 + \cot\gamma \tan E \cos\theta} \right) \leq 1}$$

That part of the area $X\,X^1Z^1Z$ that lies within the house under these conditions is unity.

For the area $X^1\,X^{11}\,Z^{11}Z^1$ we make approximations and assume the part lying within the house is given by

$$\left(1 - \frac{EY^{11}}{Y^1\,Y^{11}}\right)$$

$$= \left(\frac{1 - h^1 \cot\gamma \cos\theta - (N - n)\, w/2}{[(h^1 - h) \cot\gamma \cos\theta] + w/2 - \dfrac{w \cot\gamma \tan E \cos\theta}{1 + \cot\gamma \tan E \cos\theta}} \right)$$

For the case shown in Fig 18, I (b) in which X, X^1, X^{11} lie within the house and in which Z'' lies outside the house, we have, under the conditions

$$\boxed{h^1 \cot\gamma \cos\theta + nw/2 \leq Nw/2}$$

$$\boxed{h^1 \cot\gamma \sin\theta \leq 1}$$

$$\boxed{h^1 \cot\gamma \sin\theta + w \tan\theta > 1}$$

that for the area $X^1\,X^{11}\,Z^{11}\,Z^1$, the part that lies within the house is given by

$$\frac{1 - (\tfrac{1}{2})\,(Y^1X^1 + Y^{11}X^{11})}{(\tfrac{1}{2})\,(X^1Z^1 + X^{11}Z^{11})}$$

For the area X X^1 Z^1 Z of Fig 18 I (b), X^1 lies within the house. If Z^1 also lies within the house, that part of X X^1 Z^1Z within the house is unity.

If Z^1 lies beyond the house

ie if

$$\boxed{\left(h \cot\gamma \sin\theta + \frac{w \cot\gamma \tan E \sin\theta}{1 + \cot\gamma \tan E \cos\theta} + (w/2) \tan\theta + \frac{w \tan\theta}{1 + \cot\gamma \tan E \cos\theta}\right) > 1}$$

and

$$\boxed{\left[h + \frac{w \tan E}{1 + \cot\gamma \tan E \cos\theta}\right] \cot\gamma \cos\theta + (n - 1)\, w/2 \leq Nw/2}$$

$$\boxed{\left[h + \frac{w \tan E}{1 + \cot\gamma \tan E \cos\theta}\right] \cot\gamma \sin\theta \leq 1}$$

then that part of the area X X^1Z^1Z that lies within the house is given by

$$\frac{1 - (\tfrac{1}{2})(YX + Y^1X^1)}{(\tfrac{1}{2})(XZ + X^1Z^1)}$$

In the same way, the other possible arrangements of Fig 18, the conditions under which they arise and the corresponding value of (τF) may be arrived at. This information is brought together in a flow diagram (shown elsewhere). The notation used in this flow diagram is presented below, together with the expansion of the expressions for (τF).

Fig. 18 Flow Diagram

i) Notation

$\boxed{X \uparrow}$ $\equiv$ X beyond EF or $\boxed{h \cot\gamma \cos\theta \geq (N-n+1)w/2}$

$\boxed{X \rightarrow}$ $\equiv$ X beyond HF or $\boxed{h \cot\gamma \sin\theta > 1}$

$\boxed{X'' \uparrow}$ $\equiv$ X" beyond EF or $\boxed{h' \cot\gamma \cos\theta \geq (N-n)w/2}$

$\boxed{X'' \rightarrow}$ $\equiv$ X" beyond HF or $\boxed{h' \cot\gamma \sin\theta > 1}$

$\boxed{X' \uparrow}$ $\equiv$ X' beyond EF or $\boxed{h \cot\gamma \cos\theta + \frac{w \cot\gamma \tan\epsilon \cos\theta}{1 + \cot\gamma \tan\epsilon \cos\theta} \geq (N-n+1)w/2}$

$\boxed{X' \rightarrow}$ $\equiv$ X' beyond HF or $\boxed{h \cot\gamma \sin\theta + \frac{w \cot\gamma \tan\epsilon \sin\theta}{1 + \cot\gamma \tan\epsilon \cos\theta} > 1}$

$\boxed{Z \rightarrow}$ $\equiv$ Z beyond HF or $\boxed{h \cot\gamma \sin\theta + (w/2) \tan\theta > 1}$

$\boxed{Z' \rightarrow}$ $\equiv$ Z' beyond HF or

$$\boxed{\left[h + \frac{w \tan\epsilon}{1 + \cot\gamma \tan\epsilon \cos\theta}\right] \cot\gamma \sin\theta + \left[w/2 + \frac{w}{1 + \cot\gamma \tan\epsilon \cos\theta}\right] \tan\theta > \ell}$$

$\boxed{Z'' \rightarrow}$ $\equiv$ Z" beyond HF or $\boxed{h' \cot\gamma \sin\theta + w \tan\theta > 1}$

Fig. 18 Flow Diagram

ii) Expressions for (ΥF)

Let A_1 = Area X X' Z' Z

$A_1{}^1$ = that part of A_1 that lies within the house

B_1 = Area X' X'' Z'' Z

$B_1{}^1$ = that part of B_1 that lies within the house

$$\text{then} \quad (\Upsilon F) = \left[\frac{(\Upsilon_1)A_1{}^1 + (\Upsilon_1)^2(\Upsilon_2)B_1{}^1}{(A_1 + B_1)} \right]$$

$$A_1 + B_1 = \left[(h'-h) \cot\gamma \cos\theta + w/2 \right] \left(\tfrac{1}{2}\right) \left[(3w/2) \tan\theta \right]$$

$$A_1 = \left[\frac{w \cot\gamma \tan\epsilon \cos\theta}{1 + \cot\gamma \tan\epsilon \cos\theta} \right] \left(\tfrac{1}{2}\right) \left[(w/2) \tan\theta + \frac{w \tan\theta}{1 + \cot\gamma \tan\epsilon \cos\theta} \right]$$

$$\boxed{A_1 = w^2/2 \left[\frac{(3/2) \tan\theta + \cot\gamma \tan\epsilon (\cos\theta + \sin\theta)}{1 + \cot\gamma \tan\epsilon \cos\theta} \right]}$$

$$B_1 = \left[(h'-h) \cot\gamma \cos\theta + w/2 - \frac{w \cot\gamma \tan\epsilon \cos\theta}{1 + \cot\gamma \tan\epsilon \cos\theta} \right] \left(\tfrac{1}{2}\right) \left[w \tan\theta + \frac{w \tan\theta}{(1 + \cot\gamma \tan\epsilon \cos\theta)} \right]$$

$$\boxed{B_1 = (w/2)\tan \left[\frac{2 + \cot\gamma \tan\epsilon \cos\theta}{1 + \cot\gamma \tan\epsilon \cos\theta} \right] \left[(h'-h)\cot\gamma \cos\theta + (w/2) - \frac{w \cot\gamma \tan\epsilon \cos\theta}{1 + \cot\gamma \tan\epsilon \cos\theta} \right]}$$

when Area X X' Z' Z lies within the parallels GE, FH with X within the house and X' outside the house

$$A_1{}^1 = A_1 \left[1 - \frac{EY'}{YY'} \right]$$

$$A_1{}^1 = A_1 \left[1 - \frac{\left(h \cot\gamma \cos\theta + \dfrac{w \cot\gamma \tan\epsilon \cos\theta}{1 + \cot\gamma \tan\epsilon \cos\theta} - (N-n+1)\,(w/2) \right)}{\dfrac{w \cot\gamma \tan\epsilon \cos\theta}{1 + \cot\gamma \tan\epsilon \cos\theta}} \right]$$

when Area X' X'' Z'' Z' lies within the parallels GE, FH with X' within the house and X'' outside the house

$$B_1{}^1 \simeq B_1 \left[1 - \frac{E\,Y''}{Y'\,Y''} \right]$$

$$B_1{}^1 = B_1 \left[\frac{1 - [h' \cot\gamma \cos\theta - (N-n)\,(w/2)]}{\left[(h'-h)\cot\gamma\cos\theta + w/2 - \frac{w\cot\gamma\tan\epsilon\cos\theta}{1 + \cot\gamma\tan\epsilon\cos\theta} \right]} \right]$$

when Area X X' Z' Z lies within the parallels GH, EF and both X and X' lies within the house, whilst Z lies inside the house but Z' outside the house, we assume

$$A_1{}^1 \simeq A_1$$

when Area X X' Z' Z lies with the parallels GH, EF and both X X' lie within the house, but both Z and Z' lie outside the house

$$A_1{}^1 = A_1 \left[\frac{1 - \frac{1}{2}(YX + Y'X')}{\frac{1}{2}(XZ + X'Z')} \right]$$

$$A_1{}^1 = A_1 \left(\frac{21 - \left(2h\cot\gamma\sin\theta + \frac{w\cot\gamma\tan\epsilon\sin\theta}{1 + \cot\gamma\tan\epsilon\cos\theta}\right)}{(w/2)\tan\theta + \left[(w/2)\tan\theta + \frac{w\tan\theta}{1 + \cot\gamma\tan\epsilon\cos\theta}\right]} \right)$$

$$= A_1 \left(\frac{2(1 - h\cot\gamma\sin\theta)(1 + \cot\gamma\tan\epsilon\cos\theta) + w\cot\gamma\tan\epsilon\sin\theta}{w\tan\theta(1 + \cot\gamma\tan\epsilon\cos\theta) + w\tan\theta} \right)$$

when Area X' X'' Z'' Z' lies within the parallels GH, EF and both X' X'' lies within the house, with Z' within the house but with Z'' outside the house, we assume

$$B_1{}^1 \simeq B_1$$

when the Area X' X'' Z'' Z' lies within the parallels GH, EF, with both X', X'' inside the house, but both Z', Z'' lie outside the house we have

$$B_1{}^1 = B_1 \left[\frac{1 - (\frac{1}{2})(Y'X' + Y''X'')}{(\frac{1}{2})(X'Z' + X''Z'')} \right]$$

$$B_1{}^1 = B_1 \left(\frac{21 - \left[(h'+h)\cot\gamma\sin\theta + \frac{w\cot\gamma\tan\epsilon\sin\theta}{1 + \cot\gamma\tan\epsilon\cos\theta}\right]}{w\tan\theta + \left[(w/2)\tan\theta + \frac{w\tan\theta}{1 + \cot\gamma\tan\epsilon\cos\theta}\right]} \right)$$

$$B_1{}^1 = B_1 \left(\frac{[21 - (h'+h)\cot\gamma\sin\theta][1 + \cot\gamma\tan\epsilon\cos\theta] + w\cot\gamma\tan\epsilon\sin\theta}{(3w/2)\tan\theta[1 + \cot\gamma\tan\epsilon\cos\theta] + w\tan\theta} \right)$$

Alternative expressions (arrived at through flow diagram of Fig. 18 for (F)

1. $$(\Upsilon F) = \frac{(\Upsilon_1) A_1}{A_1 + B_1} \left[\frac{1 - (\frac{1}{2})\ (XY + X'Y')}{(\frac{1}{2})\ (XZ + X'Z')} \right]$$

2. $$(\Upsilon F) = \frac{(\Upsilon_1) A_1}{A_1 + B_1}$$

3. $$(\Upsilon F) \quad \frac{(\Upsilon_1) A_1}{A_1 + B_1} \left[1 - \frac{EY'}{YY'} \right]$$

4. $$(\Upsilon F) = \frac{(\Upsilon_1) A_1}{A_1 + B_1} + \frac{(\Upsilon_1)^2 (\Upsilon_2) B_1}{A_1 + B_1} \left[1 - \frac{EY''}{Y'Y''} \right]$$

5. $$(\Upsilon F) = \frac{(\Upsilon_1) A_1}{A_1 + B_1} \left[\frac{1-(\frac{1}{2})(XY+X'Y')}{(\frac{1}{2})(XZ+X'Z')} \right] + \frac{(\Upsilon_1)^2 (\Upsilon_2) B_1}{A_1 + B_1} \left[\frac{1-(\frac{1}{2})(X'Y'+X''Y'')}{(\frac{1}{2})(X'Z' + X''Z'')} \right]$$

6. $$(\Upsilon F) = \frac{(\Upsilon_1) A_1 + (\Upsilon_1)^2 (\Upsilon_2)\ B_1}{A_1 + B_1}$$

9.1.3.(C) $\boxed{\gamma < E}$ $\boxed{5w/2(h^1-h) \geq \text{Cot}\,\gamma > 3w/2(h^1-h)}$

$\boxed{n \text{ is odd}}$ ie faces the sun

The situation is shown in cross section in Fig 19 and in plan in Fig 20.

From our previous analysis:

$$H^1 = h + \frac{w \tan E}{1 + \text{Cot}\,\gamma \tan E \cos\theta}$$

$$x^1 = \frac{w}{1 + \text{Cot}\,\gamma \tan E \cos\theta}$$

And in the same way we obtain:

$$H^{11} = h + \frac{2 w \tan E}{1 + \text{Cot}\,\gamma \tan E \cos\theta}$$

$$x^{11} = \frac{2 w}{1 + \text{Cot}\,\gamma \,\text{Tan}\, E \cos\theta}$$

The transmitting area of the (n^{th}) roof surface is still that found in the previous section 9.1.3.(b).

The transmission coefficient appropriate to the area XX^1Z^1Z on the ground is (τ_1)

The transmission coefficient appropriate to the area $XX^{11}X^{11}Z^1$ on the ground is $(\tau)^2.(\tau_2)$

The transmission coefficient appropriate to the area $X^{11}X^{111}Z^{111}Z^{11}$ on the ground is $(\tau_1)^3 . (\tau_2)^2$

In Fig 20,

the distance $X\,Z = (w/2) \tan\theta$

the distance $X\,{}^1Z^1 = (w/2 + x^1) \tan\theta = \left[w/2 + \frac{w}{1 + \cot\gamma \tan E \cos\theta}\right] \tan\theta$

the distance $X\,{}^{11}X^{11} = (w/2 + 2\,x^1) \tan\theta = \left[w/2 + \frac{2\,w}{1 + \cot\gamma \tan E \cos\theta}\right] \tan\theta$

the distance $X^{111}\,Z^{111} = w \tan\theta$

The location of the points X, X^1, X^{11}, X^{111}, Z, Z^1, Z^{11}, Z^{11} from the sides GH, GE are as follows:

$$X. = h \cot\gamma \cos\theta + (n-1)w/2$$
$$h \cot\gamma \,\text{Sin}\,\theta$$

$$X^1 = \left[h + \frac{w \tan \epsilon}{1 + \text{Cot}\,\gamma \tan E \cos\theta}\right] \text{Cot}\,\gamma \,\text{Cos}\,\theta + (n-1)w/2$$

$$\left[h + \frac{w \tan E}{1 + \cot\gamma \tan E \cos\theta} \right] \text{Cot}\,\gamma\, \text{Sin}\,\theta$$

$$X^{11} \equiv \left[h + \frac{2w \tan E}{1 + \cot\gamma \tan E \cos\theta} \right] \text{Cot}\,\gamma\, \text{Sin}\,\theta$$

$$\left[h + \frac{2w \tan E}{1 \cot\gamma \tan E \cos\theta} \right] \text{Cot}\,\gamma\, \text{Sin}\,\theta$$

$$X^{111} \equiv \begin{array}{l} h^{1} \cot\gamma\, \text{Cos}\,\theta + n(w/2) \\ h^{1} \cot\gamma\, \text{Sin}\,\theta \end{array}$$

$$Z = \begin{array}{l} h \cot\gamma \cos\theta + (n-1)w/2 \\ h \cot\gamma \sin\theta + (w/2) \tan\theta \end{array}$$

$$Z^{1} \equiv \left[h + \frac{w \tan E}{1 + \cot\gamma \tan E \cos\theta} \right] \text{Cot}\,\gamma\, \text{Cos}\,\theta + (n-1)w/2$$

$$\left[h + \frac{w \tan E}{1 + \cot\gamma \tan E \cos\theta} \right] \text{Cot}\,\gamma\, \text{Sin}\,\theta + \left[w/2 + \frac{w}{1 + \cot\gamma \tan E \cos\theta} \right] \tan\theta$$

$$Z^{11} \equiv \left[h + \frac{2\,w \tan E}{1 + \cot\gamma \tan E \cos\theta} \right] \text{Cot}\,\gamma \cos\theta + (n-1)w/2$$

$$\left[h + \frac{2\,w \tan E}{1 + \cot\gamma \tan E \cos\theta} \right] \text{Cot}\,\gamma\, \text{Sin}\,\theta + \left[w/2 + \frac{2\,w}{1 + \cot\gamma \tan E \cos\theta} \right] \tan\theta$$

$$Z^{111} \equiv \begin{array}{l} h^{1} \cot\gamma \cos\theta + nw/2 \\ h^{1} \cot\gamma \sin\theta + w \tan\theta \end{array}$$

The evaluation of (TF) follows a similar procedure to that shown in the previous section 9.1.3.(b). We set up a flow diagram corresponding to the alternative configurations possible for Fig 20 and adopt a similar notation.

$\boxed{X \quad \uparrow}$ = X beyond EF or $\boxed{h \cot\gamma\cos\theta \geq (N-n+1)\, w/2}$

$\boxed{X \quad \rightarrow}$ = X beyond EF or $\overline{\underline{h \cot\gamma \sin\theta > 1}}$

$\boxed{X^{111} \quad \uparrow}$ = X^{111} beyond EF Z or $\boxed{h^{1} \cot\gamma \cos\theta \geq (N-n)\, W/2}$

$\boxed{X^{111} \quad \rightarrow}$ = X^{111} beyond EF or $\boxed{h^{1} \cot\gamma \sin\theta > 1}$

$\boxed{X^{11} \uparrow}$ = X^{11} beyond EF or

$$\boxed{h \cot\gamma \cos\theta + \frac{2\,w \cot\gamma \tan E \cos\theta}{1 + \cot\gamma \tan E \cos\theta} \geq (N-n+1)w/2}$$

$\boxed{X^{11} \rightarrow}$ = X^{11} beyond HF or

$$\boxed{h \cot\gamma \sin\theta + \frac{2\,w \cot\gamma \tan E \cos\theta}{1 + \cot\gamma \tan E \cos\theta} > 1}$$

$\boxed{X^{1} \uparrow}$ = X^{1} beyond EF or

$$\boxed{h \cot\gamma \cos\theta + \frac{w \cot\gamma \tan E \cos\theta}{1 + \cot\gamma \tan E \cos\theta} \geq (N-n+1)w/2}$$

$\boxed{X^{1} \rightarrow}$ = X^{1} beyond HF or

$$\boxed{h \cot\gamma \sin\theta + \frac{w \cot\gamma \tan E \cos\theta}{1 + \cot\gamma \tan E \cos\theta} > 1}$$

$\boxed{Z \rightarrow}$ = Z beyond HF or $\boxed{h \cot\gamma \sin\theta + (w/2) \tan\theta > 1}$

$\boxed{Z^{1} \rightarrow}$ = Z^{1} beyond HF or

$$\boxed{\left[h + \frac{w \tan E}{1 + \cot\gamma \tan E \cos\theta}\right] \cot\gamma \sin\theta + \left[w/2 + \frac{w}{1 + \cot\gamma \tan E \cos\theta}\right] \tan\theta > l}$$

~~tez~~ ~~1~~

$\boxed{Z^{11} \rightarrow}$ = Z^{11} beyond HF or

$$\boxed{\left[h + \frac{2\,w \tan E}{1 + \cot\gamma \tan E \cos\theta}\right] \text{Cot}\,\gamma \sin\theta + \left[w/2 + \frac{2\,w}{1 + \cot\gamma \tan E \cos\theta}\right] \tan\theta > l}$$

~~tan > 1~~

$\boxed{Z^{111} \rightarrow}$ = Z^{111} beyond HF or $\boxed{h^{1} \cot\gamma\ \text{Sih}\,\theta + w \tan\theta > 1}$

FIG 20 FLOW DIAGRAM

Expressions for (γF)

Let A = area XX'Z'Z
A'= that part of A, that lies within the house

B = area X'X''Z''Z'
B'= that part of B, that lies within the house

C = area X''X'''Z''Z''
C'= that part of C, that lies within the house

then generally

$$(\gamma F) = \frac{(\gamma)A' + (\gamma_1)^2(\gamma_2)\,B' + (\gamma_1)^3(\gamma_2)^2 C'}{(A + B + C)}$$

and $(A + B + C) = \left[(h'-h)\cot\gamma\cos\theta + w/2\right]\left[\tfrac{1}{2}\right]\left[3(w/2)\tan\theta\right]$

$$A = \left[\frac{w\cot\gamma\tan\epsilon\cos\theta}{1 + \cot\gamma\tan\epsilon\cos\theta}\right].\left(\tfrac{1}{2}\right).\left[(w/2)\tan\theta + \frac{w\tan\theta}{1 + \cot\gamma\tan\epsilon\cos\theta}\right]$$

$$B = \left[\frac{w\cot\gamma\tan\epsilon\cos\theta}{1 + \cot\gamma\tan\epsilon\cos\theta}\right].\left(\tfrac{1}{2}\right).\left[w\tan\theta + \frac{3w\tan\theta}{1 + \cot\gamma\tan\epsilon\cos\theta}\right]$$

$$C = \left[(h-'h)\cot\gamma\cos\theta + w/2 - \frac{2w\cot\gamma\tan\epsilon\cos\theta}{1 + \cot\gamma\tan\epsilon\cos\theta}\right]\left(\tfrac{1}{2}\right)\left[(3w/2)\tan\theta + \frac{2w\tan\theta}{1+\cot\gamma\tan\epsilon\cos\theta}\right]$$

When area XX'Z'Z lies within the parallels GE, HF, with X within the house and X' outside

$$A' \simeq A\left[1 - \frac{EY'}{YY'}\right]$$

$$= A\left[1 - \frac{\left(h\cot\gamma\cos\theta + \frac{w\cot\gamma\tan\epsilon\cos\theta}{1+\cot\gamma\tan\epsilon\cos\theta} - (N-n+1)w/2\right)}{\left(\frac{w\cot\gamma\tan\epsilon\cos\theta}{1+\cot\gamma\tan\epsilon\cos\theta}\right)}\right]$$

when area X'X''Z''Z' lies within the parallels GE, HF, with X'' within the house and X'' outside

$$B' = B\left[1 - \frac{EY''}{Y'Y''}\right]$$

$$= B\left[1 - \frac{\left(h\cot\gamma\cos\theta + \frac{2w\cot\gamma\tan\epsilon\cos\theta}{1 + \cot\gamma\tan\epsilon\cos\theta} - (N-n+1)w/2\right)}{\left(\frac{w\cot\gamma\tan\epsilon\cos\theta}{1+\cot\gamma\tan\epsilon\cos\theta}\right)}\right]$$

when the area X''X'''Z'''Z'' lies within the parallels GE, HF with X'' within the house and X''' outside

$$C' = C\left[1 - \frac{E\,Y'''}{Y''\,Y'''}\right]$$

$$= C\left[1 - \frac{(h'\cot\gamma\cos\theta - (N-n)\,w/2)}{(h'-h)\cot\gamma\cos\theta + w/2 - \left(\frac{2w\cot\gamma\tan\epsilon\cos\theta}{1 + \cot\gamma\tan\epsilon\cos\theta}\right)}\right]$$

When area XX'Z'Z lies within the parellels GH, EF, with both X,X' inside the house and both Z,Z' outside the house.

$$A' = A\left[\frac{1 - (\frac{1}{2})\ (XY + X'Y')}{(\frac{1}{2})\ (XZ + X'Y')}\right]$$

$$A' = A\left[\frac{21 - \left\{2h \cot\gamma \sin\theta + \frac{w \cot\gamma \tan\epsilon \cos\theta}{1 + \cot\gamma \tan\epsilon \cos\theta}\right\}}{(w/2) \tan\theta + \left[(w/2) \tan\theta + \frac{w \tan\theta}{1 + \cot\gamma \tan\epsilon \cos\theta}\right]}\right]$$

When area X'X"Z"Z' lies within the parallels GH, EF, with both X'X" onside the house and both Z'Z" outside the house.

$$B' = B\left[\frac{1 - (\frac{1}{2})\ (X'Y'' + X''Y''')}{(\frac{1}{2})\ (X'Z'' + X''Z'')}\right]$$

$$B' = B\left[\frac{21 - \left(2h \cot\gamma \sin\theta + \frac{3w \cot\gamma \tan\epsilon \cos\theta}{1 + \cot\gamma \tan\epsilon \cos\theta}\right)}{w \tan\theta + \frac{(3w \tan\theta)}{(1 + \cot\gamma \tan\epsilon \cos\theta)}}\right]$$

When area X"X"' Z"' Z" lies within the parallels GH, EF, with both X"X"' inside the house and both Z"Z"' outside the house.

$$C' = C\left[\frac{1 - (\frac{1}{2})\ (X''Y'' + X'''Y''')}{(\frac{1}{2})\ (X''Z'' + X'''Z''')}\right]$$

$$C = C\left[\frac{21 - \left((h'+h) \cot\gamma \sin\theta + \frac{2w \cot\gamma \tan\epsilon \sin\theta}{1 + \cot\gamma \tan\epsilon \cos\theta}\right)}{(3w/2) \tan\theta + \frac{(2w \tan\theta)}{(1 + \cot\gamma \tan\epsilon \cos\theta)}}\right]$$

When area XX'Z'Z lies within the parallels GH, EF, with both XX' within the house, whilst Z lies within the house, but Z' outside the house.

then $A' \bumpeq A$

Similarly when area X'X"Z"Z' lies within the parallels GH, EF, with both X'X" within the house, whilst Z' lies within the house, but Z" lies outside.

then $B' \bumpeq B$

When area X"X"' Z"' Z" lies within the parallels GH, EF, with both X" X"' within the house, whilst Z" lies within the house, but Z"' lies outside.

then $C' \bumpeq C$

FIG 20 - FLOW DIAGRAM

ALTERNATIVE EXPRESSIONS FOR (TF) - ARRIVED AT THROUGH FLOW DIAGRAM

(1) $$(TF) = \frac{(T_1)\,A}{(A+B+C)}\left[1 - \frac{EY'}{YY'}\right]$$

(2) $$(TF) = \frac{(T_1)A + (T_1)^2(T_2)\left[1 - \frac{EY''}{Y'Y''}\right]}{(A + B + C)}$$

(3) $$(TF) = \frac{(T_1)A}{(A+B+C)}$$

(4) $$(TF) = \frac{(T_1)A}{(A+B+C)}\left[\frac{1 - (\frac{1}{2})(XY + X'Y')}{(\frac{1}{2})(XZ + X'Z')}\right]$$

(5) $$(TF) = \frac{(T_1)A}{(A+B+C)}\left[\frac{1 - \frac{1}{2}(XY + X'Y')}{(\frac{1}{2})(XZ + X'Z'}\right] + \frac{(T_1)^2(T_2)B}{(A+B+C)}\left[\frac{1 - (\frac{1}{2})(X'Y' + X''Y''}{(\frac{1}{2})(X'Z' + X''Z''}\right]$$

(6) $$(TF) = \frac{(T_1)A}{(A+B+C)} + \frac{(T_1)^2(T_2)B}{(A+B+C)}\left[\frac{1-(\frac{1}{2})(X'Y' + X''Y'')}{(\frac{1}{2})(X'Z' + X''Z''}\right]$$

(7) $$(TF) = \frac{(T_1)A}{(A+B+C)} + \frac{(T_1^{\,2})(T_2)B}{(A+B+C)}$$

(8) $$(TF) = \frac{(T_1)A}{(A+B+C)} + \frac{(T_1)^2(T_2)\,B}{(A+B+C)} + \underline{(T_1)^3(T_2)^2\ C}\left[1 - \frac{EY'''}{Y''Y'''}\right]$$

(9) $$(TF) = \frac{(T_1)A + (T_1)^2(T_2)B + (T_1)^3(T_2)^2C}{(A+B+C)}$$

(10) $$(TF) = \frac{(T_1)A + (T_1)\ (T_2)\ B}{(A+B+C)} + \frac{(T_1)^2(T_2)\ C}{(A+B+C)}\left[\frac{1 - (\frac{1}{2})(X''Y'' + X'''Y'''}{(\frac{1}{2})(X''Z'' + X'''Z''')}\right]$$

(11) $$(TF) = \frac{(T_1)\ A}{(A+B+C)} + \frac{(T_1)^2(T_2)B}{(A+B+C)}\left[\frac{1 - (\frac{1}{2})(X'Y' + X''Y''}{(\frac{1}{2})(X''Z'' + X'''Z'''}\right]$$
$$+ \frac{(T_1)^3(T_2)^2C}{(A+B+C)}\left[\frac{1 - (\frac{1}{2})(X''Y'' + X'''Z'''}{(\frac{1}{2})(X''Z'' + X'''Z'''}\right]$$

(12) $$(TF) = \frac{(T_1)A}{(A+B+C)}\left[\frac{1=(\frac{1}{2})(XY + X'Y')}{(\frac{1}{2})(XZ + X'Z'}\right] + \frac{(T_1)\ (T_2)B}{(A+B+C)}\left[\frac{1 - (\frac{1}{2})(X'Y' + X''Y''}{(\frac{1}{2})(X'Z' + X''Y''}\right]$$

(d) $\boxed{\gamma < E}$ $\boxed{7w/2(h'-h) \geqslant \cot\gamma > 5w/2(h'-h)}$

$\boxed{(n) \text{ is odd}}$ = ie faces the sun

With the solar elevation in this range, we find that for typical houses (values of w, h', h) that the solar elevation is of the order 5°. The values of I_γ at these elevations are small, and any contribution to the energy received by the floor is neglected for values of $\cot\gamma > 5w/2(h'-h)$

9.1.4 Case. Roof surface n = n ; $\boxed{\theta = (t - A_{pn})}$

$$90^\circ \geq \theta \geq 0^\circ$$

ie in the nth roof surface when the up-sun surface is n = N
[The cases n = 1 n = N already discussed are excluded]

9.1.4 (a)

$\boxed{\gamma \geq E}$ ie $\boxed{w/2(h' - h) \geq \cot \gamma}$

$\boxed{(n) \text{ is even}}$ ie faces the sun.

For the case when $\theta = 0$, the situation is shown in cross-section in Fig 15(b). In plan, for the more general case ($\theta \neq 0$), the situation is that of Fig 11, when the points G and E, F and H, are interchanged.

We find now in Fig 11, that X will be outside the house if

$$\boxed{h \cot \gamma \cos \theta \geq w/2}$$

or

$$\boxed{h \cot \gamma \sin \theta \geq 1}$$

and for this situation, we take (ΥF) = 0

If X' lies outside the house

then

$$\boxed{h' \cot \gamma \cos \theta \geq (n-1) \quad w/2}$$

or $\boxed{h' \cot \gamma \quad \sin \theta \geq 1}$

and we again take (ΥF) = 0

With both X, X' inside the house,

$$(\Upsilon) = (\Upsilon_2)$$

and (F) follows from the analysis given for the similar case when (n = 1) so that

$$(\Upsilon F) = (\Upsilon_2)\left[1 - \frac{(h' + h) \quad \cot \gamma \quad \sin \theta}{21}\right]$$

$\boxed{\gamma \geq \epsilon}$ $\boxed{(n) \text{ is odd}}$ ie faces away from the sun.

We again refer to Figs. 11 and 15(b).

If X lies outside the house, we take $(\Upsilon F) = 0$

If X' lies outside the house we take $(\Upsilon F) = 0$

ie $(\Upsilon_1 F) = 0$ if any of the following conditions are met

$$\boxed{h \cot \gamma \cos \theta \geq (n-1)\, w/2}$$

$$\boxed{h \cot \gamma \sin \theta \geq 1}$$

$$\boxed{h' \cot \gamma \cos \theta \geq n\, w/2}$$

$$\boxed{h' \cot \gamma \sin \theta \geq 1}$$

with both X, X' lying within the floor of the house,

$$(\Upsilon) = (\Upsilon_1)$$

and (F) follows from the similar case when the upsun surface is $(n = 1)$ and

$$(\Upsilon F) = (\Upsilon_1)\left[1 - \frac{(h' + h) \cot \gamma \sin \theta}{21}\right]$$

9.1.4. (b) $\boxed{\gamma < \epsilon}$ $\boxed{3w/2(h'-h) \geq \cot\gamma > w/2(h'-h)}$

$\boxed{n \text{ is even}}$ ie faces the sun – upsun surface is n = N

We are again concerned with shading of the transmitting roof surface. The transmitting area $(\acute{a})$ is again that found in section 9.1.3.(b). and is again subject to the proviso that the maximum value of $(\acute{a})$ is

$$(\acute{a}) = (a) = 1.\ (w/2)\ \sec\epsilon$$

In section, the situation is that of Fig. 15(c) and in the general case of $\theta = 0$ is again essentially that of Fig. 16. However since the upsun surface is now (n=N) rather than (n=1), the locations of the points X'', X', X, Z'', Z', Z, from the sides of the house read somewhat differently from the case of section 9.1.3.(b). and

$$X = \begin{array}{l} h\cot\gamma\cos\theta + (N-n)\ w/2 \\ h\cot\gamma\sin\theta \end{array}$$

$$X' = \begin{array}{l} \left[h + \frac{w\tan\epsilon}{1 + \cot\gamma\tan\epsilon\cos\theta}\right]\left[\cot\gamma\cos\theta\right] + (N-n)\ w/2 \\ \left[h + \frac{w\tan\epsilon}{1 + \cot\gamma\tan\epsilon\cos\theta}\right]\left[\cot\gamma\sin\theta\right] \end{array}$$

$$X'' = \begin{array}{l} h'\cot\gamma\cos\theta + (N-n+1)\ w/2 \\ h'\cot\gamma\sin\theta \end{array}$$

$$Z = \begin{array}{l} h\cot\gamma\cos\theta + (N-n)\ w/2 \\ h\cot\gamma\sin\theta + (w/2)\tan\theta \end{array}$$

$$Z' = \begin{array}{l} \left[h + \frac{w\tan E}{1 + \cot\gamma\tan E\cos\theta}\right]\left[\cot\gamma\cos\theta\right] + (N-n)\ w/2 \\ \left[\frac{h + w\tan E}{1 + \cot\gamma\tan E\cos\theta}\right]\left[\cot\gamma\sin\theta\right] + \left[w/2 + \frac{w}{1 + \cot\gamma\tan E\cos\theta}\right]\tan\theta \end{array}$$

$$Z'' = \begin{array}{l} h'\cot\gamma\cos\theta + (N-n+1)\ w/2 \\ h'\cot\gamma\sin\theta + w\tan\theta \end{array}$$

To evaluate the proportions of the areas XX'Z'Z, X'X''Z''Z', that fall within the floor area of the house, and the quantities (τF), the flow diagram appropriate Fig. 18 [and the corresponding expressions for (γF)] still hold, provided we modify suitably the criteria specified in the notation eg

$\boxed{X\uparrow}$ = X beyond GH or $\boxed{h\cot\gamma\cos\theta + (N-n)\ w/2 \geq Nw/2}$

$\boxed{X \rightarrow}$ = X beyond HF or $\boxed{h \cot\gamma \sin\theta > 1}$

$\boxed{X'' \uparrow}$ = X'' beyond GH or $\boxed{h' \cot\gamma \cos\theta + (N-n+1)\, w/2 \geq NW/2}$

$\boxed{X'' \rightarrow}$ = X'' beyond HF or $\boxed{h' \cot\gamma \sin\theta > 1}$

$\boxed{X' \uparrow}$ = X' beyond GH or

$$\boxed{h \cot\gamma \cos\theta + \frac{w \cot\gamma \tan E \cos\theta}{1 + \cot\gamma \tan E \cos\theta} + (N-n)\, w/2 \geq NW/2}$$

$\boxed{X' \rightarrow}$ = X' beyond HF or

$$\boxed{h \cot\gamma \sin\theta + \frac{w \cot\gamma \tan E \sin\theta}{1 + \cot\gamma \tan E \cos\theta} > 1}$$

$\boxed{Z \rightarrow}$ = Z beyond HF or $\boxed{h \cot\gamma \sin\theta + (w/2) \tan\theta > 1}$

$\boxed{Z' \rightarrow}$ = Z' beyond HF or

$$\boxed{\left[h + \frac{w \tan E}{1 + \cot\gamma \tan E \cos\theta}\right] \cot\gamma \sin\theta + \left[w/2 + \frac{w}{1 + \cot\gamma \tan E \cos\theta}\right] \tan\theta > 1}$$

$\boxed{Z'' \rightarrow}$ = Z'' beyond HF or $\boxed{h' \cot\gamma \sin\theta + w \tan\theta > 1}$

9.1.4(c) $\boxed{\gamma < \epsilon}$

$$\boxed{5w/2(h'-h) \geq \cot\gamma > 3w/2 \quad (h'-h)}$$

$\boxed{\text{n is even}}$ ie faces the sun - upsun surface is $n = N$

In essentials, the problem is again that of Figs. 19 and 20, if the points G,E: F,H, are interchanged. The locations of the points X, X', X'', X''', Z, Z', Z'', Z''' from the sides of the house differ slightly from these found in section 9.1.3(c) since the upsun surface is now ($n=N$) rather than ($n=1$).

The flow diagram corresponding Fig. 20 and the corresponding expressions for (γF) will again apply, provided we modify the criteria set out in the notation as follows.

$\boxed{X \uparrow}$ = X beyond GH or $\boxed{h \cot\gamma \cos\theta + (N-n)w/2 \geq Nw/2}$

$\boxed{X \rightarrow}$ = X beyond HF or $\boxed{h \cot\gamma \sin\theta > 1}$

$\boxed{X''' \uparrow}$ = X''' beyond GH or $\boxed{h' \cot\gamma \cos\theta + (N-n+1)w/2 \geq Nw/2}$

$\boxed{X''' \rightarrow}$ = X''' beyond HF or $\boxed{h' \cot\gamma \sin\theta > 1}$

$\boxed{X'' \uparrow}$ = X'' beyond GH or

$$\boxed{h \cot\gamma \cos\theta + \frac{2w \cot\gamma \tan\epsilon \cos\theta}{1 + \cot\gamma \tan\epsilon \cos\theta} + (N-n)w/2 \geq Nw/2}$$

$\boxed{X'' \rightarrow}$ = X'' beyond HF or

$$\boxed{h \cot\gamma \sin\theta \frac{+ 2w \cot\gamma \tan E \sin\theta}{1 + \cot\gamma \tan E \cos\theta} > 1}$$

$\boxed{X'\uparrow}$ = X' beyond GH or

$$\boxed{h \cot\gamma \cos\theta + \frac{w \cot\gamma \tan E \cos\theta}{1 + \cot\gamma \tan E \cos\theta} + (N-n)\, w/2 \geqslant Nw/2}$$

$\boxed{X'\rightarrow}$ = X' beyond HF or

$$\boxed{h \cot\gamma \sin\theta + \frac{w \cot\gamma \tan E \sin\theta}{1 + \cot\gamma \tan E \cos\theta} > 1}$$

$\boxed{Z \rightarrow}$ = Z beyond HF or $\boxed{h \cot\gamma \sin\theta + (w/2) \tan\theta > 1}$

$\boxed{Z'\rightarrow}$ = Z' beyond HF or

$$\boxed{\left[h + \frac{w \tan E}{1 + \cot\gamma \tan E \cos\theta}\right] \cot\gamma \sin\theta + \left[w/2 + \frac{w}{1 + \cot\gamma \tan E \cos\theta}\ \cancel{\tan}\right] \tan\theta > l}$$

$\boxed{Z''\rightarrow}$ = Z''' beyond HF or

$$\boxed{\left[h + \frac{2w \tan E}{1 + \cot\gamma \tan E \cos\theta}\right] \cot\gamma \sin\theta + \left[w/2 + \frac{2w}{1 + \cot\gamma \tan E \cos\theta}\right] \tan\theta > l}$$

$\boxed{Z'''\rightarrow}$ = Z''' beyond HF or $\boxed{h' \cot\gamma \sin\theta + w \tan\theta > 1}$

9.1.4(d) $\boxed{\gamma < E}$

$$\boxed{\cot\gamma > 5w/2\,(h'-h)}$$

For the reasons set out in 9.1.3.(d), the energy received by the floor is neglected for these low values of solar elevation (γ).

10. EFFECTS DUE TO REFLECTION OF DIRECT RADIATION

10.1 EXTERNAL REFLECTION BETWEEN OPPOSING ROOF SURFACES

We assume roof slopes of less than 45° and so avoid the complication of multiple reflection between opposing roof surfaces.

The incident beam, the reflected beam and the normal to the reflecting beam at the point of reflection all lie in the same plane. In the limiting case of $\theta = 0$, this plane is also the vertical plane and this situation is shown in Fig. 21. The effect of reflection is simply to rotate the incident beam (of solar elevation γ) through an angle of $(180^\circ + 2E - 2\gamma)$, so that the elevation of the reflected beam (γ_R) becomes

$$\gamma_R = (2E - \gamma)$$

The more general case ($\theta \neq 0$) is shown in Fig. 22. A unit sphere is drawn centred on P, the point of incidence on the receiving surface. IP is the incident ray. The normal to the reflecting surface ABC is indicated by PN. The reflected ray is PR. The vertical through P is shown by PV.

The incident ray IP, the normal to the reflecting plane PN and the reflected ray PR all still lie in the same plane PINR, but this is no longer the plane of incidence AIVP. To locate the point R of the reflected ray, both in azimuth and elevation above the horizontal, we have, in the spherical triangle IVR to calculate the angle V and the angular distance of the Side VR.

For greater simplicity, the situation is set out again in Fig. 23. Let RVN = ψ

Using standard forms, we have in the spherical triangle IVN

$$\frac{\mathrm{Sin}\,(90 - \gamma + E)}{\mathrm{Sin\ VNI}} = \frac{\mathrm{Sin}\,\beta}{\mathrm{Sin}\,(180 - \theta)}$$

$$\text{and Sin VNI} = \frac{\sin\,(90 - \gamma + E)\ \sin\,(180^\circ - \theta)}{\sin\beta}$$

from which we may obtain the angle VNI

In the spherical triangle RNV

Angle RVN = 180° - angle VNI = (ξ) say

Again using standard forms we have

$$\tan\left(\frac{\psi + R}{2}\right) = \cos\left(\frac{\beta - E}{2}\right) \sec\frac{(\beta + E)}{2} \cot\frac{\xi}{2}$$

$$\tan\left(\frac{\psi - R}{2}\right) = \sin\left(\frac{\beta - E}{2}\right) \operatorname{cosec}\left(\frac{\beta + E}{2}\right) \cot\frac{\xi}{2}$$

from which ψ and R may be found.

[(β) is of course the angle of incidence onto the reflecting surface and the expression to determine (β) is given in Ag. Mem 277]

also we have

$$\sin\frac{VR}{2} = \cos\frac{\xi}{2} \operatorname{cosec}\left(\frac{\psi - R}{2}\right) \sin\left(\frac{\beta - E}{2}\right)$$

from which VR may be found.

The azimuth (θ_R) of the reflected beam (when the incident beam has an azimuth (θ) onto the reflecting surface) is now simply

$$\theta_R = \psi$$

The angular elevation (γ_R) (above the horizontal) of the reflected beam [when the incident beam is from an elevation (γ), onto a plane of tilt (E), the azimuth of the incident beam being (θ)], is now

$$\gamma_R = - (90^\circ - VR)$$

or,

$$\gamma_R = VR - 90^\circ$$

A negative value for γ_R implies an apparent origin below the horizontal. Such radiation cannot reach the floor of the house, except by reflection at an internal roof surface.

Except in the limiting case of ($\theta = 0$), it is apparent that there is no very simple relationship between the elevation and azimuth of the incident beam and the elevation and azimuth of the corresponding reflected beam reaching the opposing roof surface.

Fig 24 shows the general case (for high solar elevations ie $\gamma \geq E$) in which incident radiation of intensity I_γ is reflected from the surface (n = 2) onto the opposing surface (n = 3). The azimuth of the incoming radiation is α and the up-sun surface is (n = 1).

The general expression for the direct radiation intercepted by the surface (n = 2) is

$$I_\gamma \cos\beta_2 \quad R \quad a$$

where the suffix (2) denotes we consider the surface (n = 2)

R is the relative duration of sunshine

(a) is the overall reflecting area of the surface (n = 2)

The radiation reflected from the surface (n = 2) if we ignore absorbtion in the glass, is taken as

$$I_\gamma \cos\beta_2 . \; R. \; a. \; (1 - \tau_2)$$

where (τ_2) is the transmission coefficient for direct radiation incident at an angle (β_2) from a solar elevation (γ).

The amount of this reflected radiation intercepted by the surface (n = 3) ie from an elevation (γ_R) and azimuth (ϕ) is given by

$$I_\gamma \cos\beta_2 . \; R. \; a. \; (1 - \tau_2) \; F$$

where (F) = Angle Factor

$$= \frac{\text{Area } DPP^1D^{11}}{\text{Area } (a)}$$

$$(F) = \frac{\text{Area } DPP^1D^{11}}{\text{Area } MDD^1M^1}$$

To calculate the area DPP^1D^{11} we need to locate the point P. Fig. 25 shows the detailed geometry, together with some necessary construction lines.

Let angle PGL = K

Then in the triangles GLN, GLP we have

$$GL = NG \sin E = PG \cos K$$

$$NL = NG \cos E$$

$$PL = PG \sin K$$

In the triangle NLP we have

$$NL = PL \cos \theta$$

so that

$$NL = NG \cos E = PG \sin K \cos \theta$$

Hence $$\frac{NG}{PG} = \frac{\cos K}{\sin E} = \frac{\sin K \cos \theta}{\cos E}$$

or,

$$\boxed{\tan K = \cot E \cos \theta}$$

In the triangle MPG

$$\frac{PG}{\sin \gamma_R} = \frac{MG}{\sin (180^\circ - \gamma_R - 90^\circ + K)} = \left[\frac{MG}{\cos(\gamma_R - K)}\right] = \frac{MP}{\cos K}$$

so that $$PG = \frac{MG \sin \gamma_R}{\cos(\gamma_R - K)}$$

But $$NG = \frac{PG \cos K}{\sin E} = \frac{MG \sin \gamma_R}{\cos(\gamma_R - K)} \cdot \frac{\cos K}{\sin E}$$

and $MG = w/\cos \theta$

Hence $$NG = \frac{w \cos K \sin \gamma_R}{\cos \theta \sin E \cos(\gamma_R - K)}$$

also $DR = DQ - RQ = DQ - NG$

$$\boxed{DR = \frac{w}{2 \cos E} - \frac{w \cos K \sin \gamma_R}{\cos \theta \sin E \cos(\gamma_R - K)}}$$

Now
$$\begin{aligned} RP &= RN - PN \\ &= QG - PN \\ &= w \tan \theta - NL \tan \theta \\ &= w \tan \theta - \frac{GL \tan \theta}{\tan E} \end{aligned}$$

also GL = BP and in the triangles MGC, MBP

$$BP = GC . \frac{MP}{GC} = (h^1 - h) . \frac{MG \cos K}{\cos(\gamma_R - K)} . \frac{1}{MC}$$

and $BP = \dfrac{(h^1 - h) . \cos K . \cos \gamma_R}{\cos(\gamma_R - K)}$

So that

$$RP = w \tan \theta - \frac{(h^1 - h) \cos K \cos \gamma_R}{\cos(\gamma_R - K)} . \frac{\tan \theta}{\tan E}$$

$$\boxed{RP = w \tan \theta - \frac{(w/2) \cos K \cos \gamma_R \tan \theta}{\cos(\gamma_R - K)}}$$

Hence the Angle Factor (F) $= \dfrac{\text{Area } DPP^1D^{11}}{\text{Area } MDD^1M^1}$

or

$$\boxed{(F) = \frac{\left[1 - w \tan \theta - \dfrac{(w/2) \cos K \cos \gamma_R \tan \theta}{\cos(\gamma_R - K)}\right]\left[\dfrac{(w/2)}{\cos E} - \dfrac{w \cos K \sin \gamma_R}{\operatorname{Cos} \theta \sin E \cos(\gamma_R - K)}\right]}{1 (w/2) \sec E}}$$

We note that

$\boxed{\text{If } RP \geq 1}$, then (F) = 0

or,

$\boxed{\text{If } DR > (w/2)/\cos E}$, then (F) = 0

To find the direct radiation reflected from the surface (n = 2) which reaches the floor of the house after passing through the surface (n = 3) we make the following assumptions

(i) $\boxed{\gamma_R \geq 0}$; $\boxed{\gamma \geq E}$ All of the surface (n = 3) transmits reflected radiation from the surface (n = 2).

This reflected radiation has

elevation γ_R

azimuth ψ

and is assumed to have intensity $I = I_\gamma \cos\beta_2 \cdot R.\ a.\ F.\ (1 - \tau_2)$

The procedure for handling this situation is set out in earlier sections.

(ii) $\boxed{\gamma_R < 0}$; $\boxed{\gamma > E}$ Such radiation has an apparent origin at ground level; we take as zero the reflected radiation from the surface $(n = 2)$ that subsequently suffers attenuation in passing through the surface $(n = 3)$ and is finally reflected down towards the ground by the surface $(n = 4)$. When the up-sun surface is $(n = 1)$, the expressions just developed will hold for the reflection of direct radiation from the surface $(n = m)$, to the surface $(n = m + 1)$, where (m) is even.

Similarly, when the upsun surface is $(n = N)$, the same expressions will hold for the reflection of direct radiation from the surface $(n = m)$ to the surface $(n = m + 1)$, where (m) is odd, provided (τ_2) is replaced by (τ_1).

Also at very high solar elevations $(\gamma \gg E)$ the possibility exists not only for reflection from the surface $(n = 2)$ to the surface $(n = 3)$ (when the upsun surface is $n = 1$), but also for reflection of direct incident radiation from the surface $(n = 3)$ to the surface $(n = 2)$. The zero value assumed for reflected direct radiation, when γ_R 0, will take account of most of these situations. If the situation still obtains where $\gamma_R \geqslant 0$, then the expressions above for (F) still hold. Provided we assign the correct value to (τ), depending on whether the initial incident radiation impinges on an odd or even numbered surface, then it is immaterial to the expressions developed for (F) whether the upsun surface is $(n = 1)$ or $(n = N)$

(iii) $\boxed{\gamma > E}$ At low solar elevations, various combinations of circumstances eg, low values of I_γ, shading of roof surfaces, large values of (β), will all combine to result in small amounts of reflected direct radiation being intercepted by an opposing roof surface.

Under these conditions, we take the reflected direct radiation reaching an opposing roof surface as zero.

10.2 REFLECTION OF DIRECT RADIATION BY AN INTERNAL ROOF SURFACE FACING THE GROUND

10.2 (a) DIRECT RADIATION ENTERS THROUGH AN END ROOF SURFACE ($n = 1$ or $n = N$)

The up-sun surface is ($n = 1$)

When $\cot\gamma \leq w/2\,(h^1 - h)$, the direct radiation reaching the ground passes through the single roof surface ($n = 1$) [cf. Fig. 13(a)]

When $\cot\gamma > w/2\,(h^1 - h)$, some direct radiation passes through more than one roof surface. In Fig. 13 (b) for example, we identify two uniform strips at the ground, associated with transmission coefficients of (τ_1) and $(\tau_1)^2$. (τ_2).

We ignore the absorbtion of radiation by the glass. Since in Fig. 13 (b) the reflection of direct radiation by the internal roof surface ($n = 2$) is down towards the ground, this reflected radiation necessarily reaches the ground up-sun of the associated transmitted beam (located by the points X^1, X^{11})

We assume that we may account for thisreflected radiation by the surface ($n = 2$) by increasing the transmission coefficient of the strip located by $X\,X^1$ from

$$(\tau_1) \quad \text{to} \quad (\tau_1) + (\tau_1)\,(1 - \tau_2)$$

and retain our earlier procedures for determining the associated quantity (F), the angle factor.

In the same way, when $5\,w/2\,(h^1 - h) \geq \cot\gamma > 3\,w/2\,(h^1 - h)$

we have the possibility of internal reflection by the surfaces ($n = 2$),

$(n = 4)$

We assume this reflection will be approximately accounted for if we increase the transmission coefficient of the strip located by $X\,X^1$ from

$$(\tau_1) \text{ to } (\tau_1) + (\tau_1)\,(1 - \tau_2)$$

and the strip located by $X^1\,X^{11}$ from

$$(\tau_1)^2\,(\tau_2) \text{ to } (\tau_1)^2\,(\tau_2) + (\tau_1)^2\,(\tau_2)\,(1 - \tau_2)$$

For lower values of Solar elevation (γ), we increase the values of the transmission coefficients assigned to the strips $X\,X^1$, X^1X^{11} $X^{11}X^{111}$ in sections 9.1.1 (d) and 9.1.1. (e) by a factor $(1 - \tau_2)$ so that they become

$$(\tau_1) \rightarrow (\tau_1) + (\tau_1)\,(1 - \tau_2)$$

$$(\tau_1)^2(\tau_2) \rightarrow (\tau_1)^2(\tau_2) + (\tau_1)^2\,(\tau_2)\,(1 - \tau_2)$$

$$(\tau_1)^3\,(\tau_2)^2 \rightarrow (\tau_1)^3\,(\tau_2)^2 + (\tau_1)^3(\tau_2)^2\,(1 - \tau_2)$$

The up-sun surface is $(n = N)$

If the surfaces $(n = 1)$ and $(n = N)$ are interchanged, the only modification required to the above expressions is that (τ_1) and (τ_2) should be interchanged.

10.2 (b) DIRECT RADIATION ENTERS THROUGH A ROOF SURFACE OTHER THAN (n = 1) OR (n = N)

The up-sun surface is $(n = 1)$

In section 9.1.3 (b), (a section which treats the case where $\gamma <$ E) we take account of reflected direct radiation by increasing the transmission coefficient in the terms involving (τ_1) alone from

$$(\tau_1) \text{ to } (\tau_1) + (\tau_1)(1 - \tau_2)$$

Similarly in section 9.1.3 (c), we increase the transmission coefficients in the terms involving (τ_1) alone from

$$(\tau_1) \text{ to } (\tau_1) + (\tau_1)\,(1 - \tau_2)$$

and the terms involving $(\tau_1)^2\,(\tau_2)$ from

$$(\tau_1)^2(\tau_2) \text{ to } (\tau_1)^2\,(\tau_2) + (\tau_1)^2(\tau_2)\,(1 - \tau_2)$$

The up-sun surface is (n = N)

Interchange of the surfaces (n = 1) and (n = N) as the up-sun surface simply requires the interchange of (τ_1) and (τ_2) in the preceeding expressions.

10.2(c) DIRECT RADIATION ENTERS THROUGH A ROOF SURFACE OTHER THAN (n = 1)

OR (n = N), but after undergoing one external reflection.

This is a perfectly possible situation, but we assume that we may neglect the contribution to the energy received by the floor of the house of direct radiation that has undergone more than one reflection.

10.3 REFLECTION OF DIRECT RADIATION BACK INTO THE HOUSE BY THE SIDE WALLS

10.3 (a) DIRECT RADIATION ENTERS THROUGH A WALL (AND IS REFLECTED BY

OPPOSING WALL SURFACES)

(F) the angle factor between the wall (through which the radiation enters) and the floor of the house, gives the proportion of the transmitted radiation which is intercepted by the floor of the house.

The remaining proportion [1 - (F)] is evidently intercepted by the walls of the house. This radiation is of known azimuth (θ) and known solar elevation (γ). and so we may readily compute a transmission coefficient for it when it arrives at the internal surface of either of the walls facing that through which it entered. Since the incoming radiation arrives from an angle (γ) above the horizontal, subsequent reflection at an internal wall surface necessarily directs the reflected radiation down toward the ground. However, not all such reflected radiation will necessarily fall within the floor of the house. It may in fact reach a further wall surface before it reaches the ground outside the house.

For a given wall which receives the initial direct solar radiation, section 7 gives the daily total of direct radiation reaching the floor of the house, through the wall as

$$\sum 60. \; I_\gamma \cos\beta \; R(\tau_W). a. P. F.$$

(where τ_W is the transmission coefficient of the wall)

The daily total of direct radiation transmitted to the opposing walls through the initial wall is

$$\sum 60.\ I_\gamma \cos\beta.\ R.(\tau_W)a.P\left[1-(F)\right]$$

We make the approximation that the daily total of direct radiation reflected back by opposing wall surfaces and intercepted by the floor of the house is

$$\sum 60.\ I_\gamma \cos\beta\ R.\ (\tau_W)\ a\ P\left[1-(F)\right]\left[1-(\tau_W^{1})\right]$$

where $(\tau_W^{'})$ is the greater of the two values of transmission coefficients calculated for the two opposing wall surfaces at any given hour.

10.3 (b) DIRECT RADIATION ENTERS THROUGH A ROOF SURFACE

(and is reflected by opposing wall surfaces)

In section 9 we have considered the direct radiation reaching the floor of the house from a roof surface. This radiation is divided into various components, depending upon the number of roof surfaces traversed before the radiation reaches ground level. It becomes necessary to consider the quantity (τF) and to evaluate the angle factor (F) [the proportion of direct radiation through the roof surface that is intercepted by the floor of the house] for each of the uniform strips of radiation, characterised variously by transmission coefficients (τ_1), $(\tau_1)^2.(\tau_2)$, $(\tau_1)^3.(\tau_2)^2$ etc

Again, we may make the point, for any one such uniform radiation strip, that if it is not intercepted by the ground within the house, then it is intercepted by the walls and that this latter component will give rise to direct radiation reflected back into the house.

Let (F_1) = Angle Factor for the first radiation strip characterised by transmission coefficient (τ_1) (say)

Let (F_2) = Angle Factor for the second radiation strip characterised by transmission coefficient $(\tau_1)^2.(\tau_2)$

Let (F_3) = Angle Factor for the third radiation strip

Characterised by transmission coefficient $(\tau_1)^3(\tau_2)^2$

[We note that there will circumstances where the terms (τ_1) and (τ_2) will be interchanged]

The daily total of direct radiation reaching the ground within the house through a given roof surface, will be of the general form

$$\sum 60\ I_\gamma \cos\beta . R.\ a.\ P.\ (\tau F)$$

where $(\tau F) = (\tau_1)(F_1) + (\tau_1)^2(\tau_2)(F_2) + (\tau_1)^3(\tau_2)^2(F_3)$

The daily total of direct radiation through a given roof surface that is intercepted by the walls is given by

$$\sum 60.\ I_\gamma .\ \cos\beta .\ R.\ a.\ P.\ [(\tau_1)(1 - F_1) + (\tau_1)^2(\tau_2)(1 - F_2) + (\tau_1)^3(\tau_2)^2(1 - F_3)]$$

Whilst it is true that radiation coming from a single roof surface at a given hour may reach only one internal wall surface, (rather than both opposing walls), which wall is so affected will change during the course of the day. Such radiation may still suffer more than one internal reflection at the walls before it reaches the ground, and not all this reflected radiation will necessarily fall within the floor of the house.

So the proposal is that we compute the transmission coefficient (τ_w) for direct radiation reaching the approx (internal) walls at each hour angle, and select the greater of these - (τ_w').

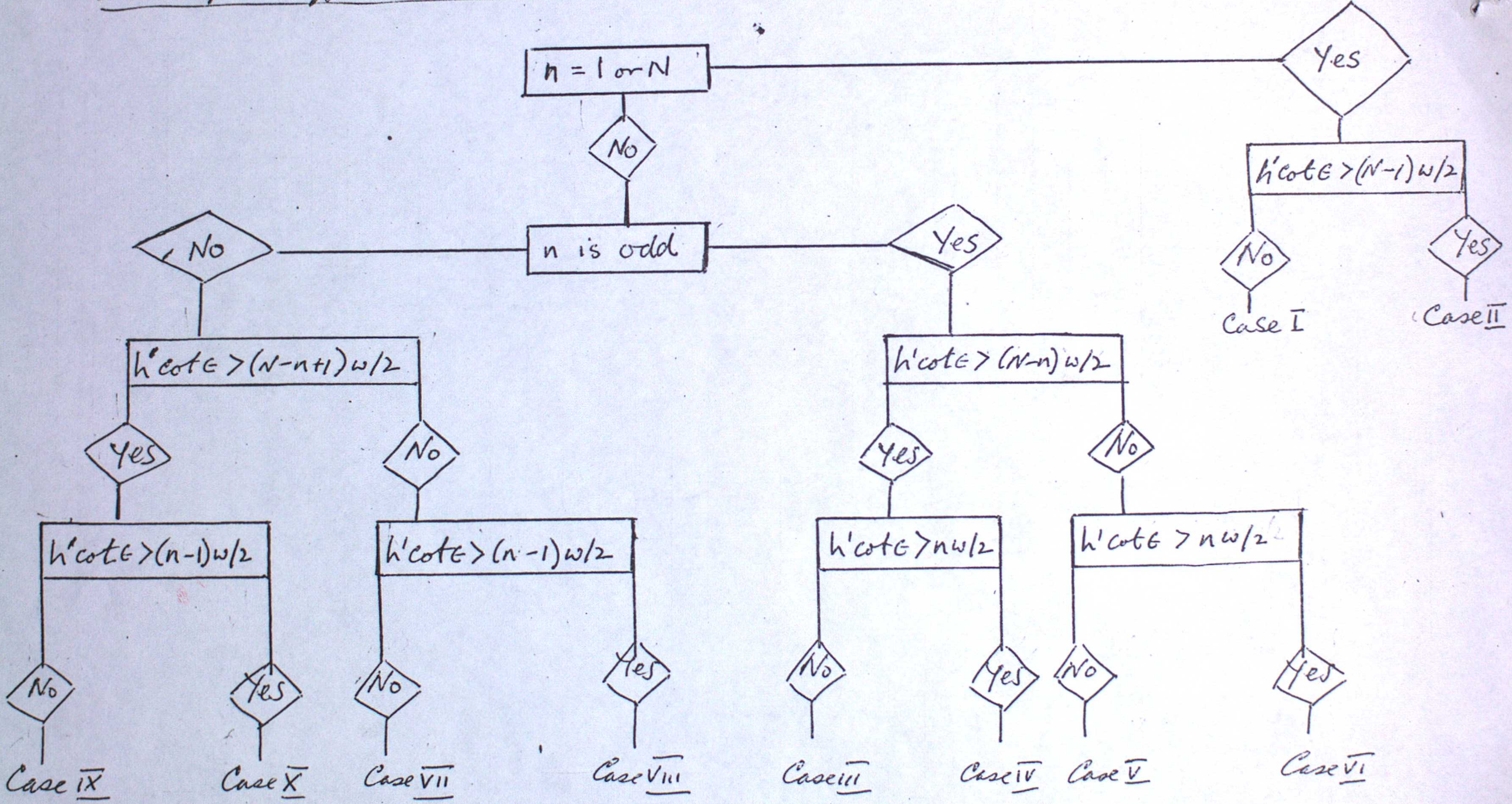
We assume that the contribution to the energy received at the floor of the house by direct radiation entering through a roof surface, being reflected by an internal wall surface and finally reaching the ground to be

$$\sum 60.\ I_\gamma .\ \cos\beta .\ R.a.P.[(\tau_1)(1 - F_1) + (\tau_1)^2(\tau_2)(1 - F_2) + (\tau_1)^3(\tau_2)^2(1 - F_3)]$$
$$[1 - (\tau_w')]$$

END

Separate sheet please

4.4.1 Suggested ~~flow~~ flow diagram to determine the width of the floor area that intercepts diffuse radiation from a roof surface



n = 1 or N
Yes
No
h'cotε > (N−1) w/2
No
Yes
Case I
Case II
n is odd
No
Yes
h'cotε > (N−n+1) w/2
h'cotε > (N−n) w/2
Yes
No
Yes
No
h'cotε > (n−1) w/2
h'cotε > (n − 1) w/2
h'cotε > nw/2
h'cotε > nw/2
No
Yes
No
Yes
No
Yes
No
Yes
Case IX
Case X
Case VII
Case VIII
Case III
Case IV
Case V
Case VI

Fig. 18 FLOW DIAGRAM SHEET 1.

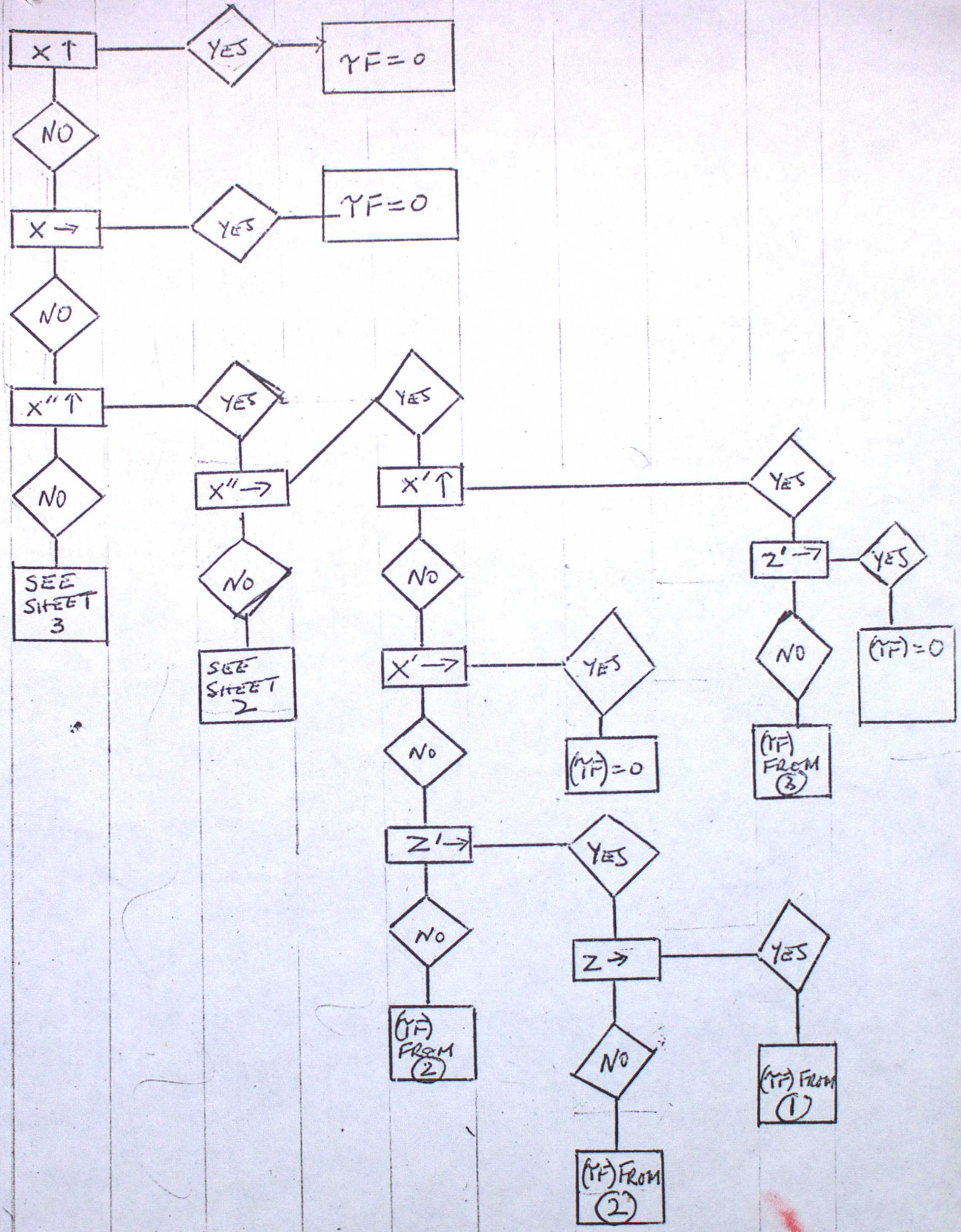

X ↑
YES
YF = 0
NO
X →
YES
YF = 0
NO
X″ ↑
YES
YES
NO
X″ →
SEE SHEET 3
X′ ↑
YES
NO
NO
Z′ →
YES
SEE SHEET 2
X′ →
YES
NO
(YF) = 0
(YF) = 0
NO
(YF) FROM ③
NO
Z′ →
YES
Z →
YES
NO
(YF) FROM ②
NO
(YF) FROM ①
(YF) FROM ②

Fig 18 Flow Diagram SHEET 2

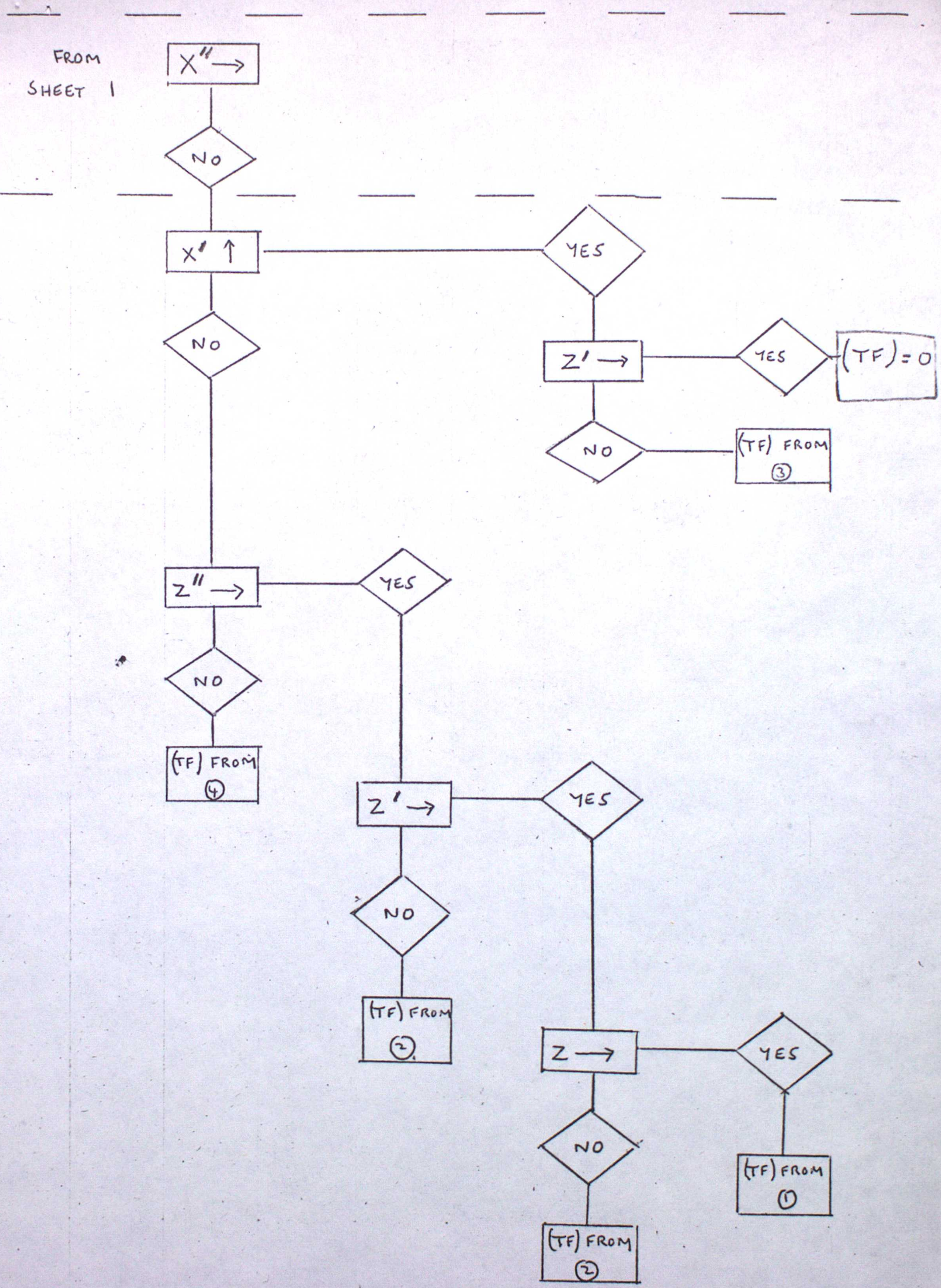

FROM
SHEET 1
X″ →
NO
X′ ↑
YES
NO
Z′ →
YES
(TF) = 0
NO
(TF) FROM ③
Z″ →
YES
NO
(TF) FROM ④
Z′ →
YES
NO
(TF) FROM ②
Z →
YES
NO
(TF) FROM ①
(TF) FROM ②

Fig 18 Flow Diagram SHEET 3

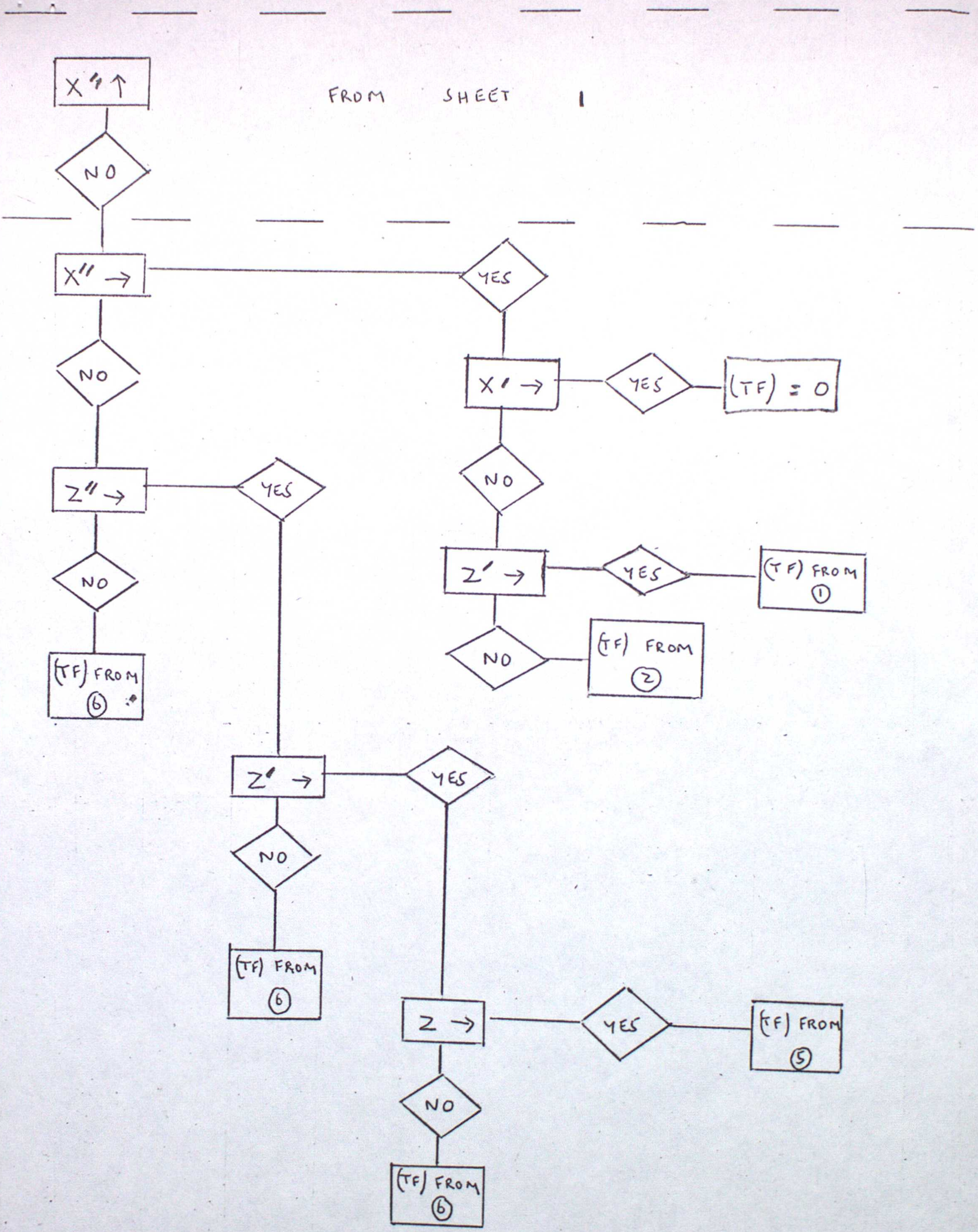

X'' ↑
FROM SHEET 1
NO
X'' →
YES
NO
X' →
YES
(TF) = 0
Z'' →
YES
NO
NO
Z' →
YES
(TF) FROM ①
NO
(TF) FROM ②
(TF) FROM ⑥
Z' →
YES
NO
(TF) FROM ⑥
Z →
YES
(TF) FROM ⑤
NO
(TF) FROM ⑥

Fig 20 Flow Diagram SHEET 1

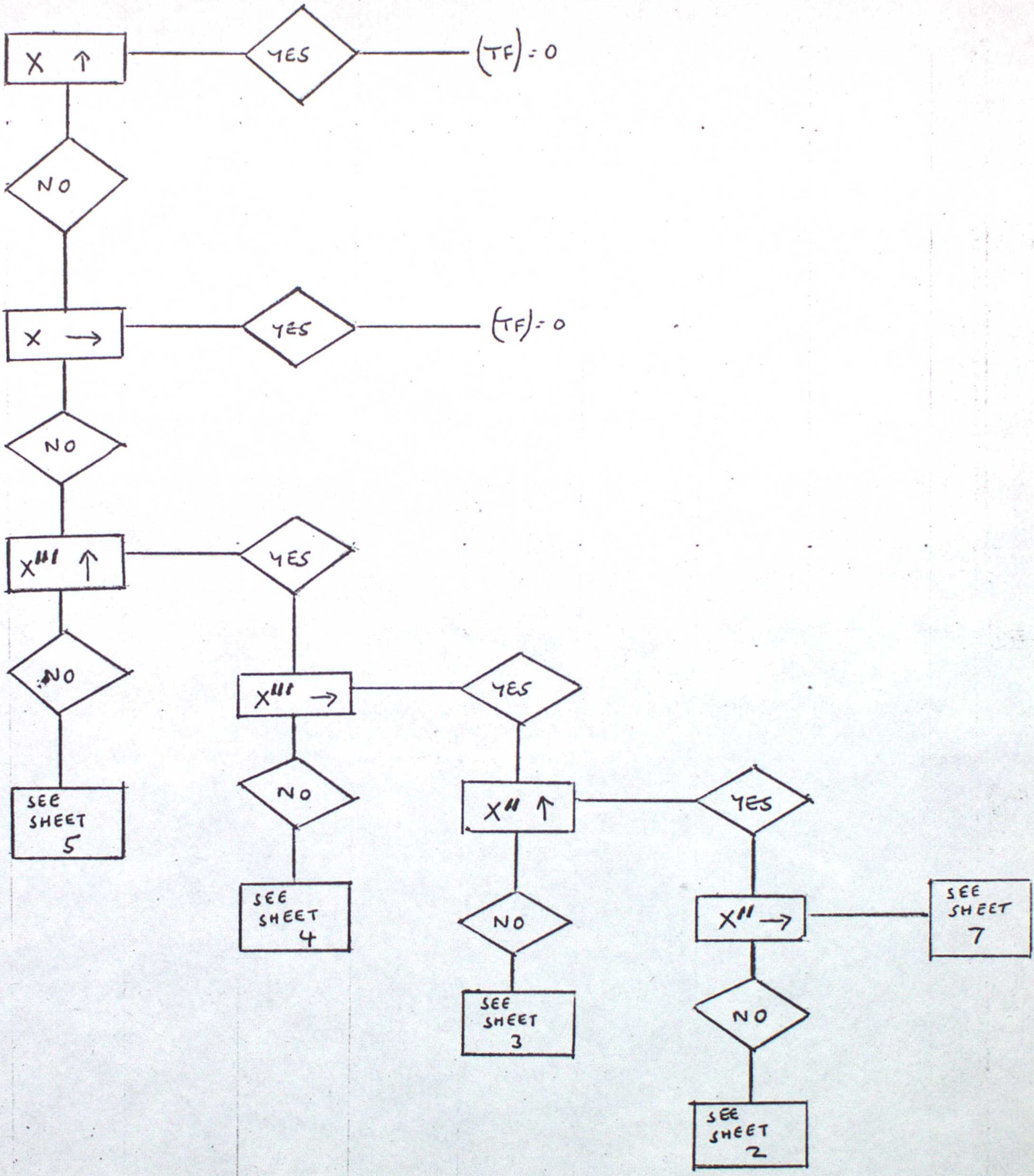

X ↑
YES
(TF) = 0
NO
X →
YES
(TF) = 0
NO
X''' ↑
YES
NO
SEE SHEET 5
X''' →
YES
NO
SEE SHEET 4
X'' ↑
YES
NO
SEE SHEET 3
X'' →
SEE SHEET 7
NO
SEE SHEET 2

Fig 20 Flow Diagram SHEET 2

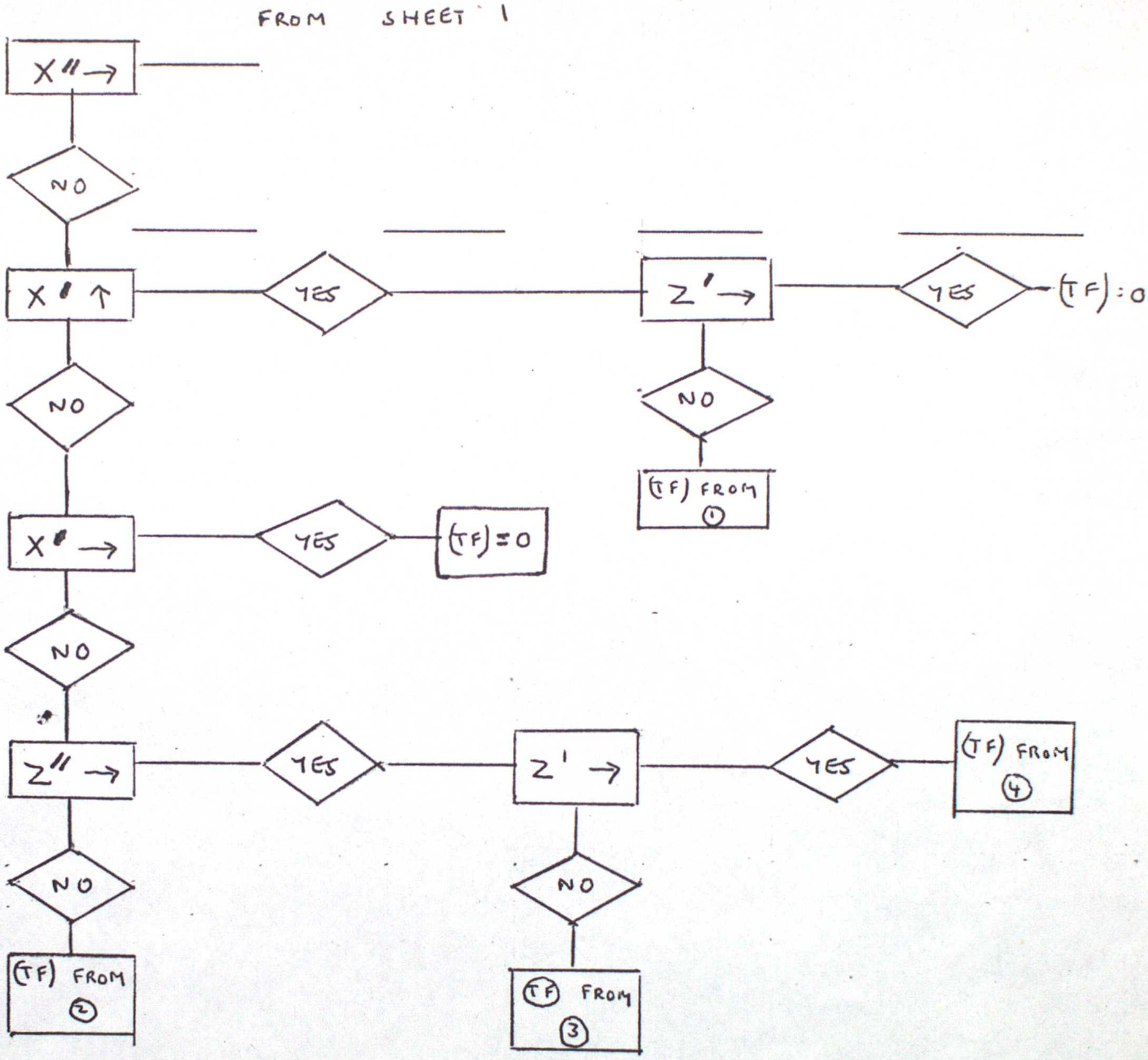

FROM SHEET 1
X″→
NO
X′↑
YES
Z′→
YES
(TF):0
NO
(TF) FROM ①
NO
X′→
YES
(TF)=0
NO
Z″→
YES
Z′→
YES
(TF) FROM ④
NO
(TF) FROM ②
NO
(TF) FROM ③

Fig 20 Flow Diagram SHEET 3

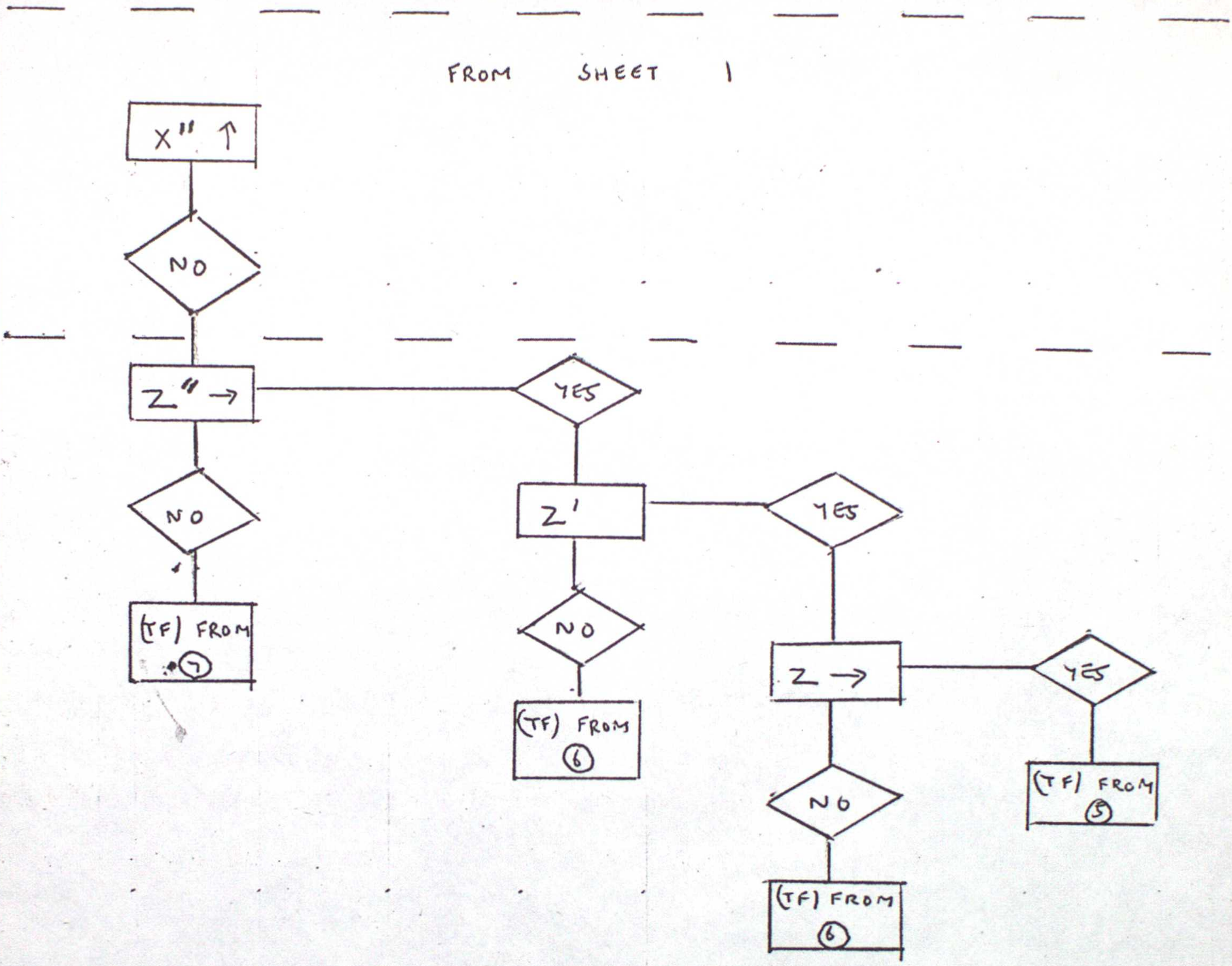

FROM SHEET 1
X'' ↑
NO
Z'' →
YES
NO
Z'
YES
(TF) FROM ⑦
NO
Z →
YES
(TF) FROM ⑥
NO
(TF) FROM ⑤
(TF) FROM ⑥

Fig. 20 FLOW Diagram SHEET 4

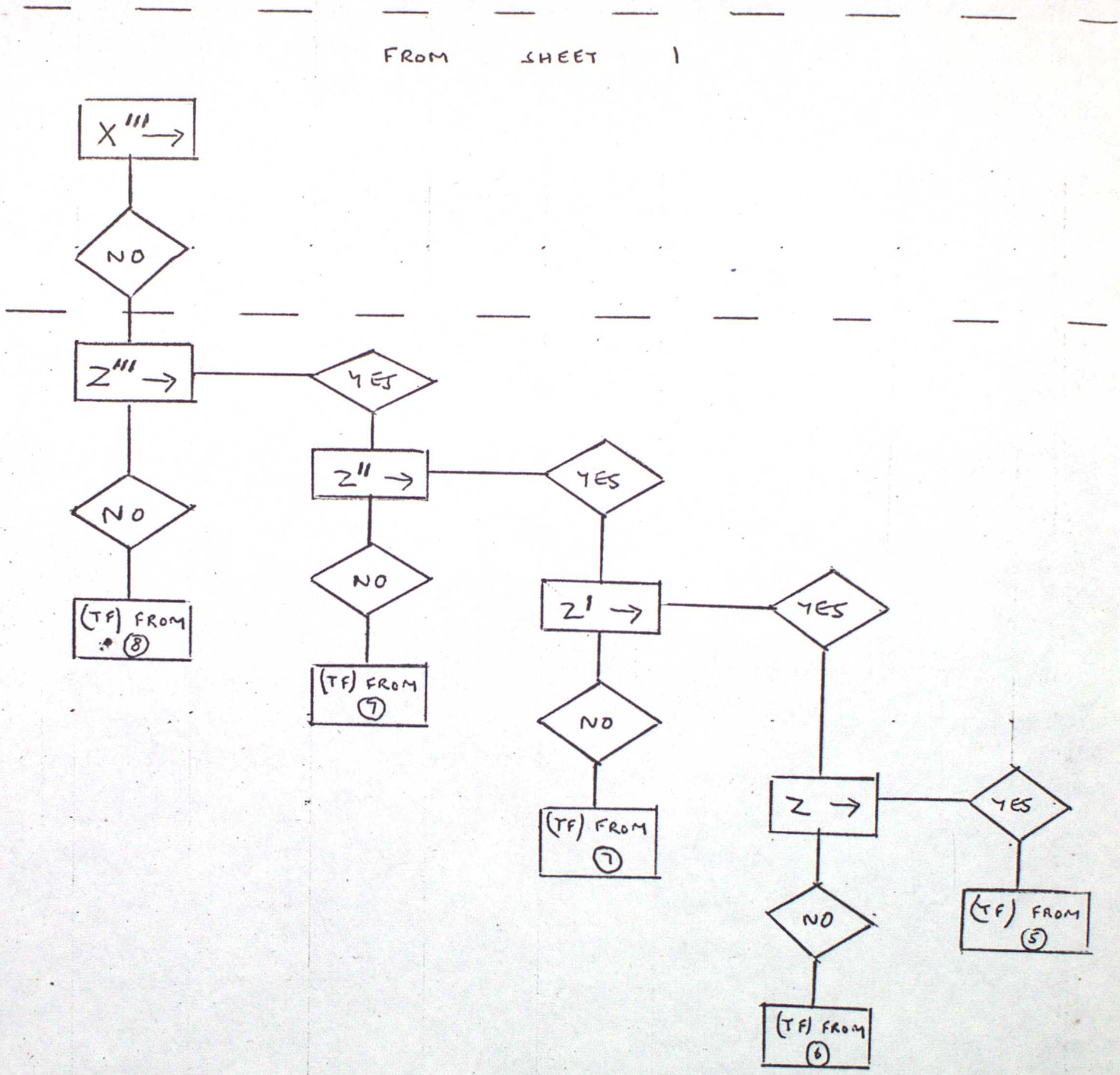

FROM SHEET 1
X''' →
NO
Z''' →
YES
NO
(TF) FROM ⑧
Z'' →
YES
NO
(TF) FROM ⑦
Z' →
YES
NO
(TF) FROM ⑦
Z →
YES
NO
(TF) FROM ⑥
(TF) FROM ⑤

Fig 20 Flow Diagram SHEET 5

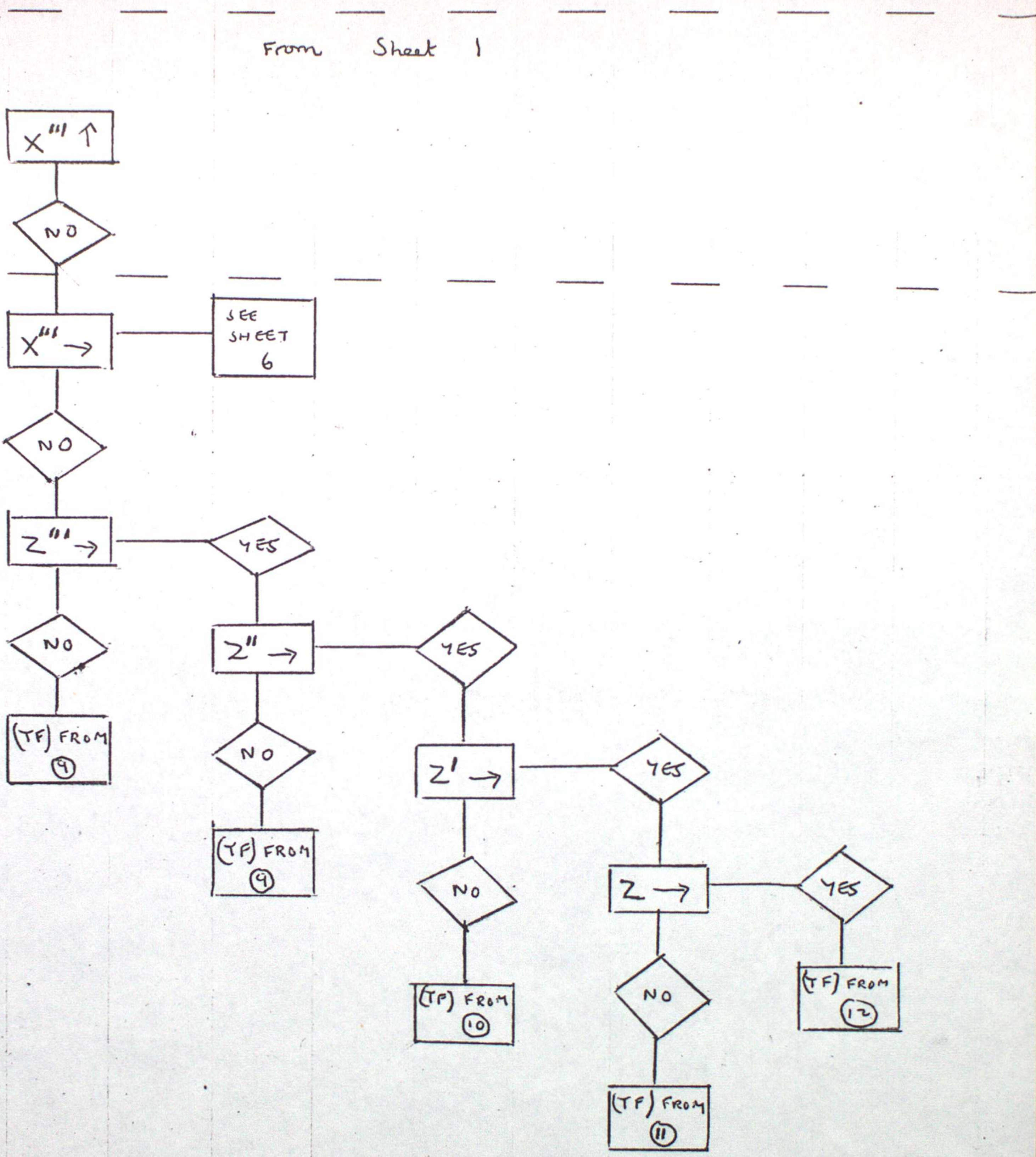

From Sheet 1
X''' ↑
NO
X''' →
SEE SHEET 6
NO
Z''' →
YES
NO
(TF) FROM ⑨
Z'' →
YES
NO
(TF) FROM ⑨
Z' →
YES
NO
(TF) FROM ⑩
Z →
YES
NO
(TF) FROM ⑪
(TF) FROM ⑫

Fig 20 Flow Diagram SHEET 6

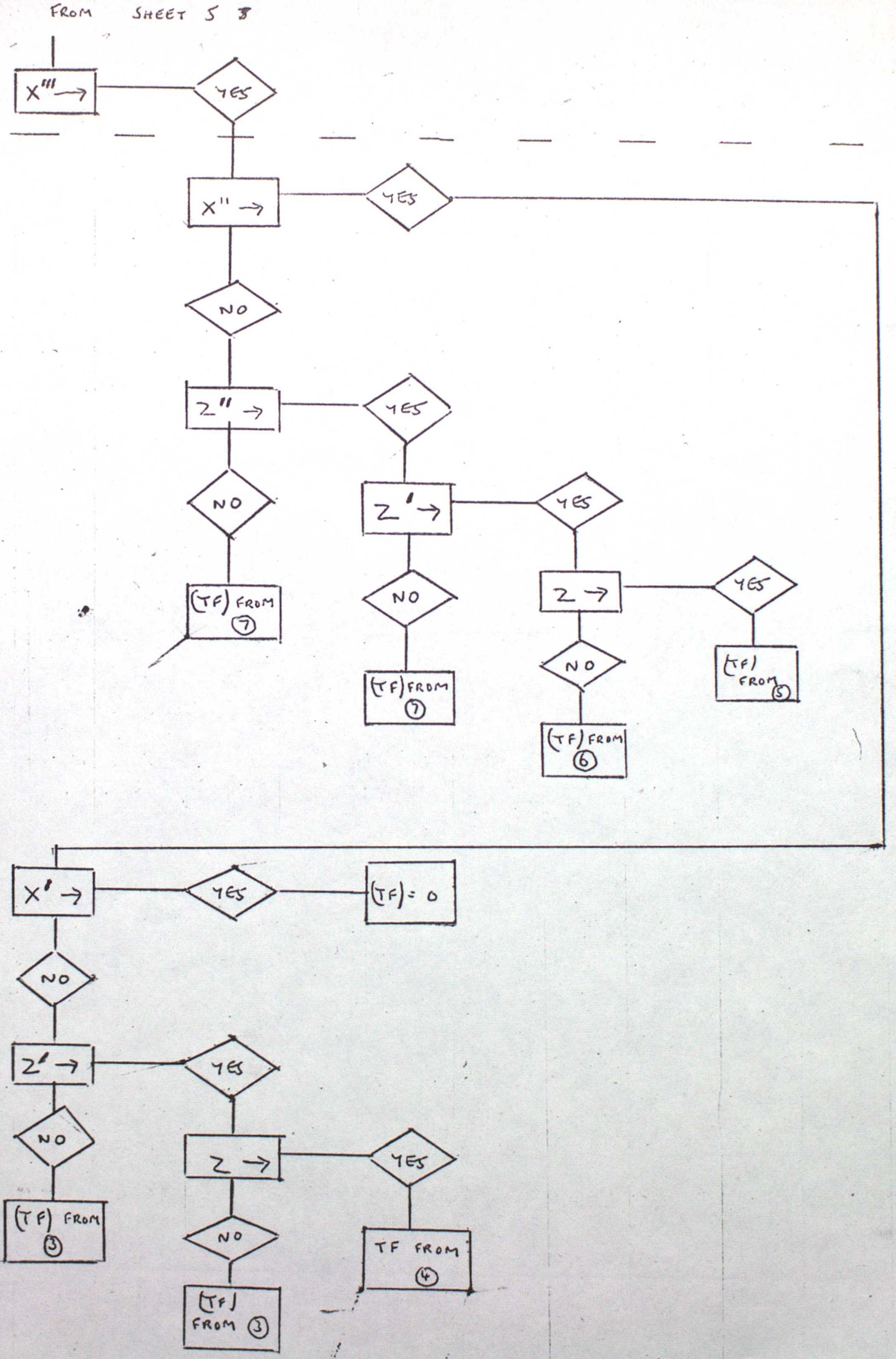

FROM SHEET 5
X''' →
YES
X'' →
YES
NO
Z'' →
YES
NO
(TF) FROM ⑦
Z' →
YES
NO
(TF) FROM ⑦
Z →
YES
NO
(TF) FROM ⑥
(TF) FROM ⑤
X' →
YES
(TF) = 0
NO
Z' →
YES
NO
(TF) FROM ③
Z →
YES
NO
(TF) FROM ③
TF FROM ④

Fig 20 Flow Diagram SHEET 7

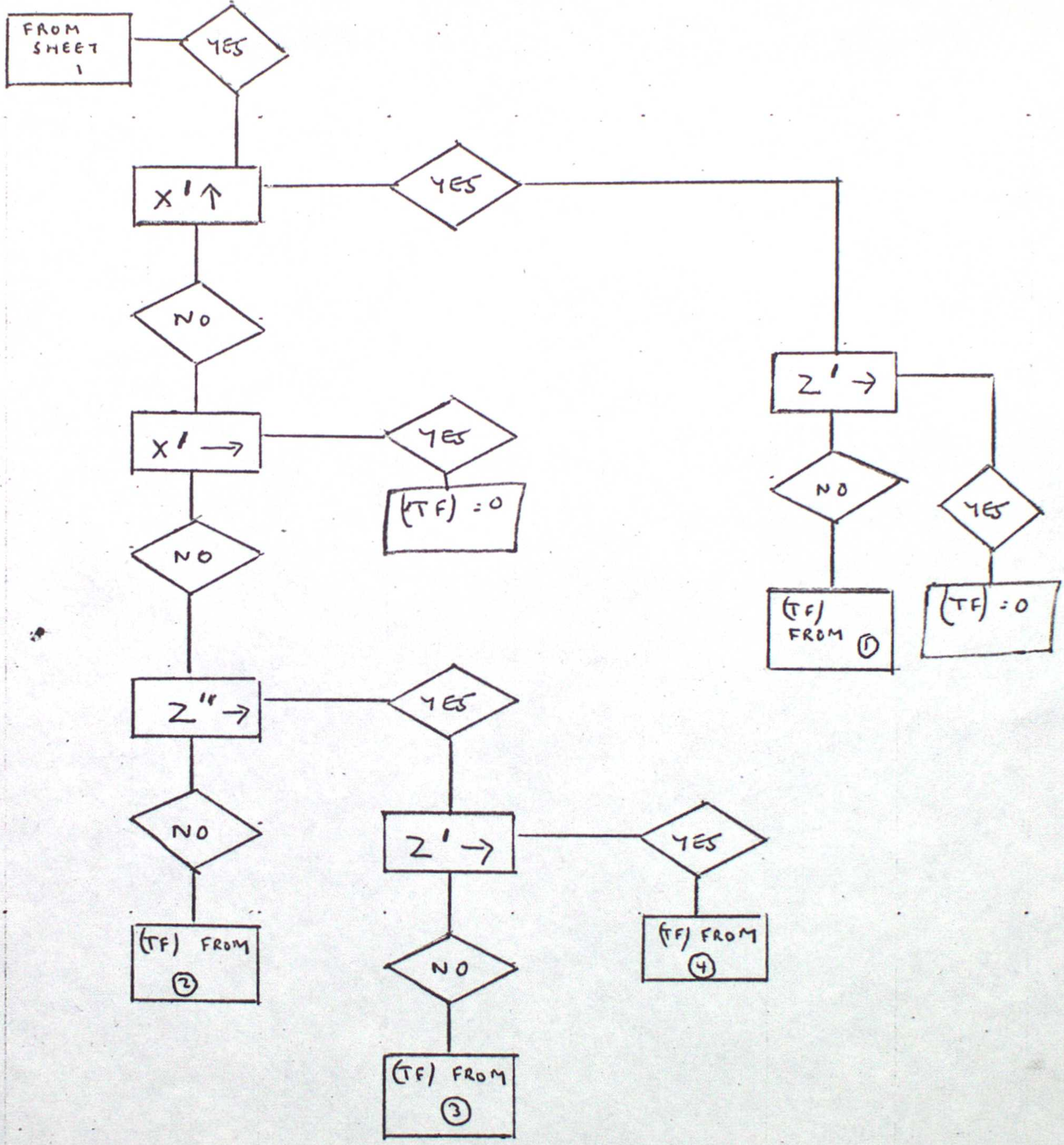

FROM SHEET 1
YES
X' ↑
YES
NO
X' →
YES
(TF) = 0
NO
Z' →
NO
YES
(TF) FROM ①
(TF) = 0
Z" →
YES
NO
Z' →
YES
(TF) FROM ②
(TF) FROM ④
NO
(TF) FROM ③

Fig. 1

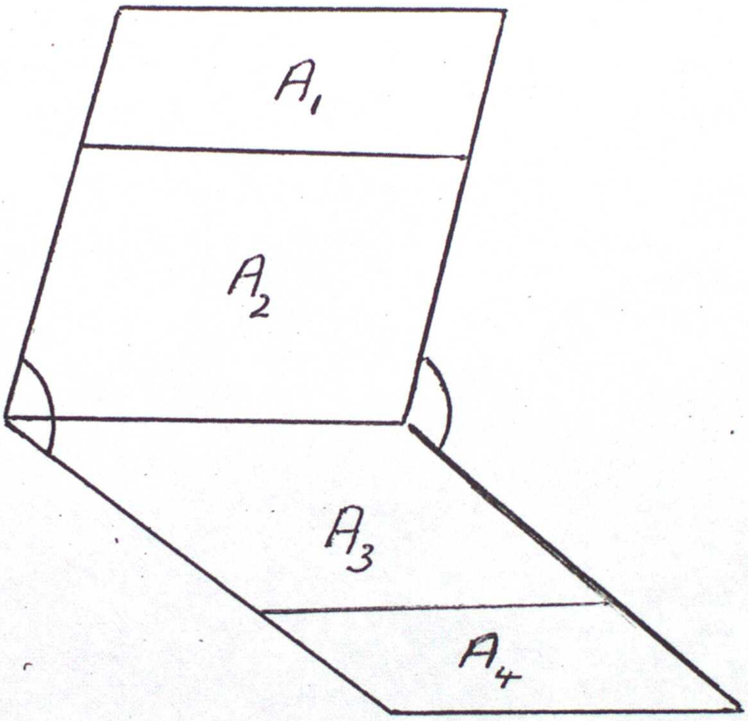
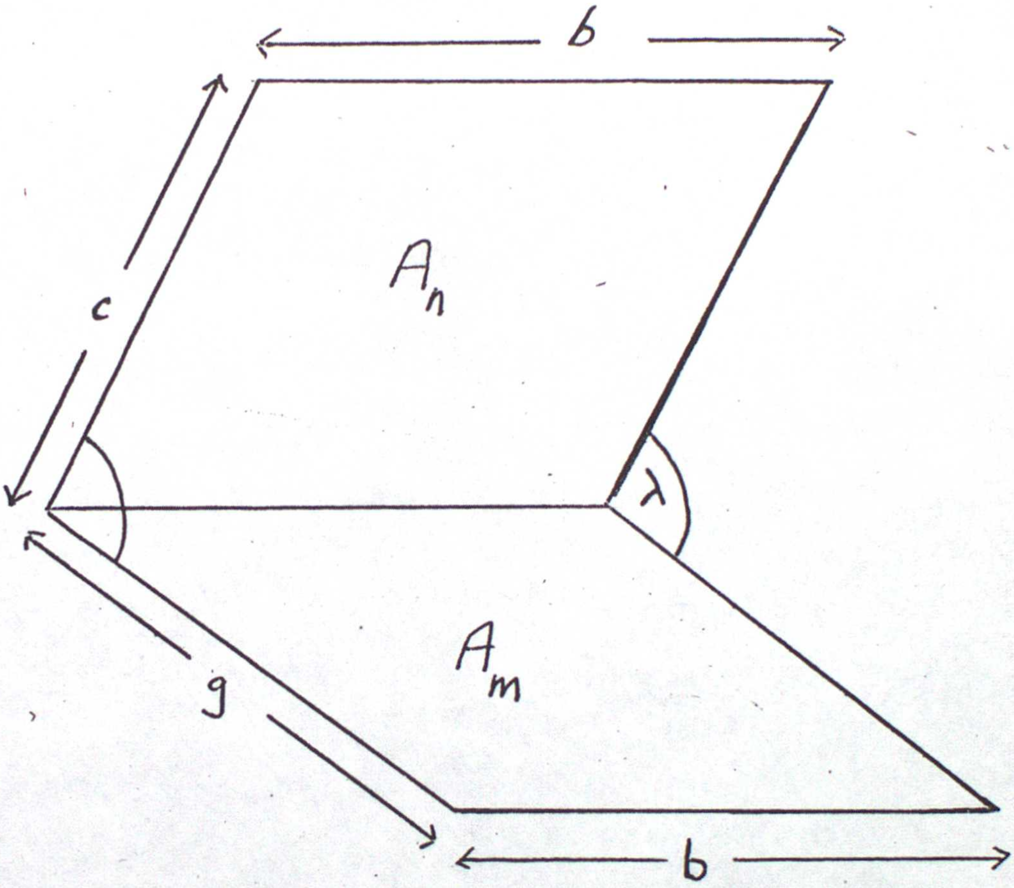


Fig. 2



b
c
A_n
λ
A_m
g
b

Fig. 3

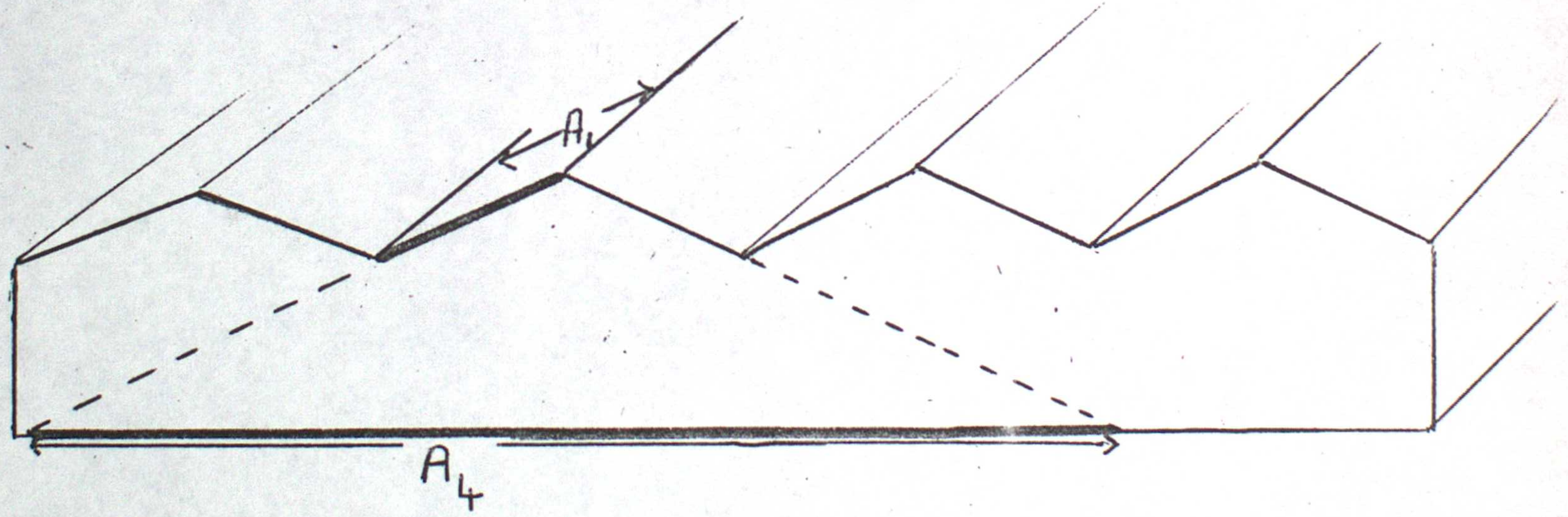


Fig. 4

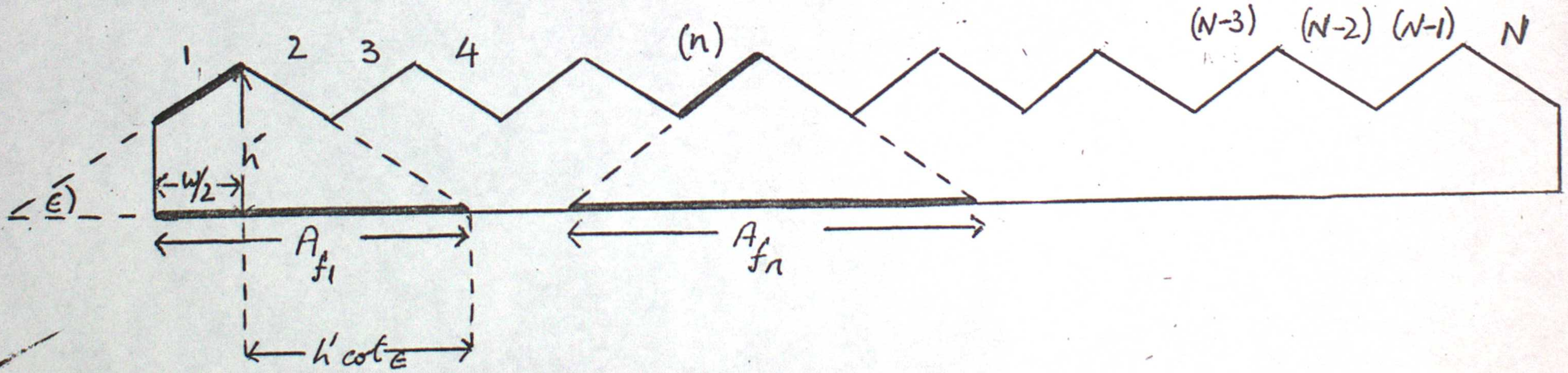
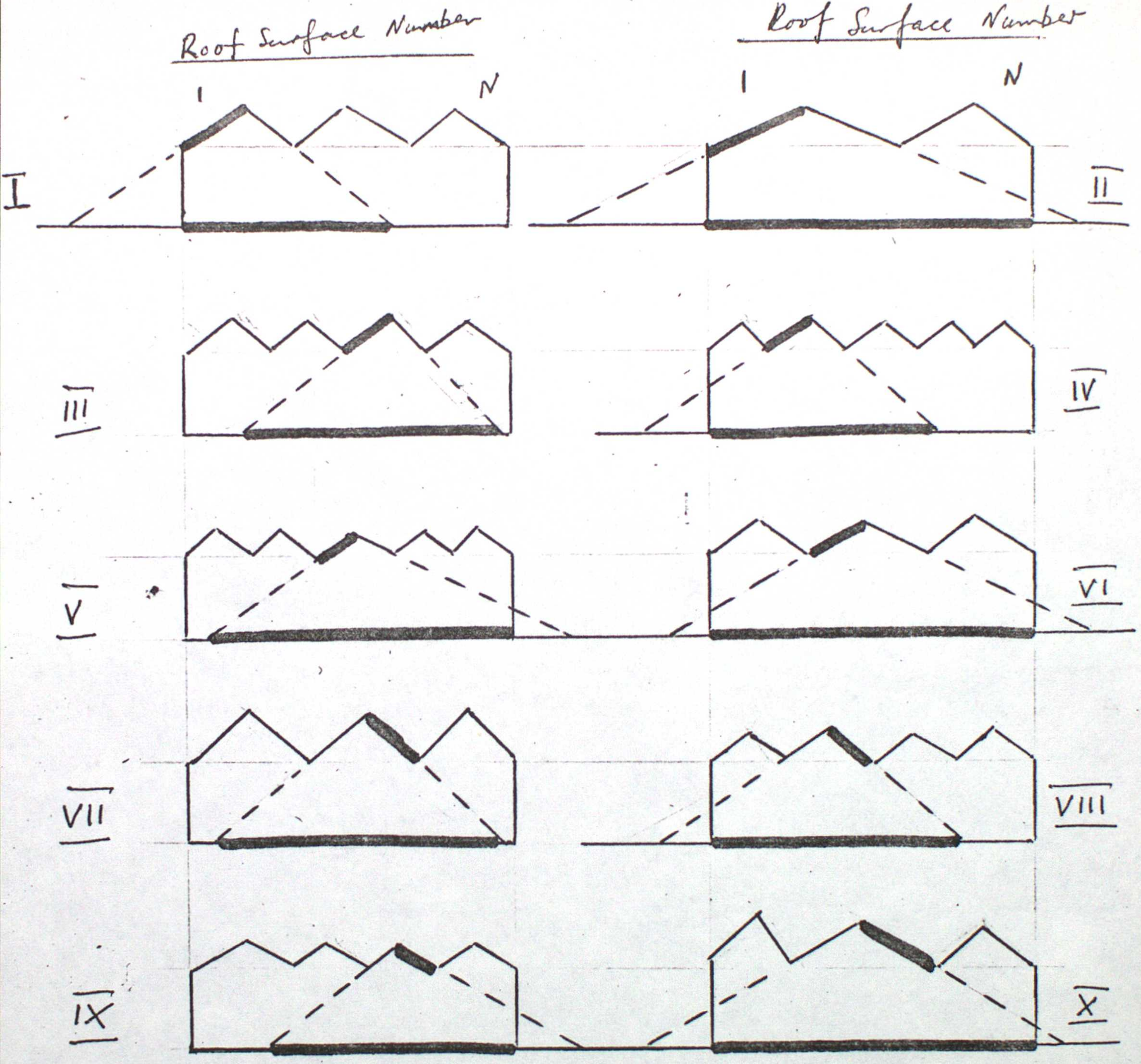
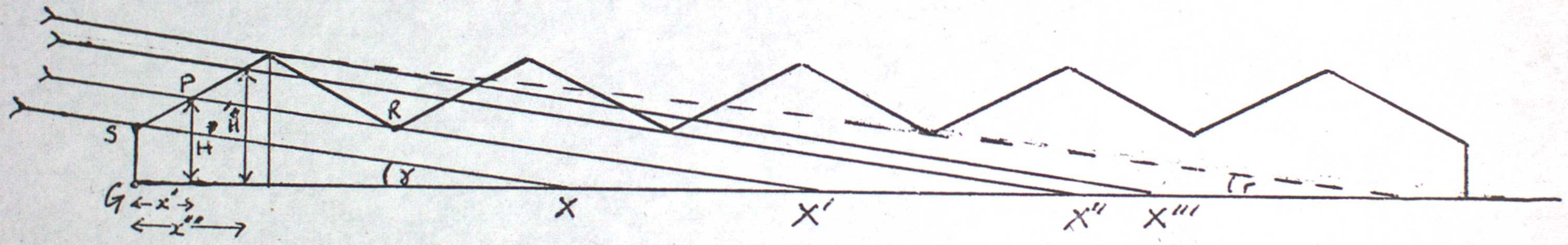

1
2
3
4
(n)
(N-3)
(N-2)
(N-1)
N
$\angle \epsilon$
$w/2$
h'
A_{f_1}
A_{f_n}
$h' \cot_\epsilon$

Fig. 5



Roof Surface Number
1
N
Roof Surface Number
1
N
I
II
III
IV
V
VI
VII
VIII
IX
X

Fig. 6


P
H'
R
S
H
γ
G
x'
x''
X
X'
X''
X'''
r

$\cot r = 5w/2(h'-h)$

Fig 7

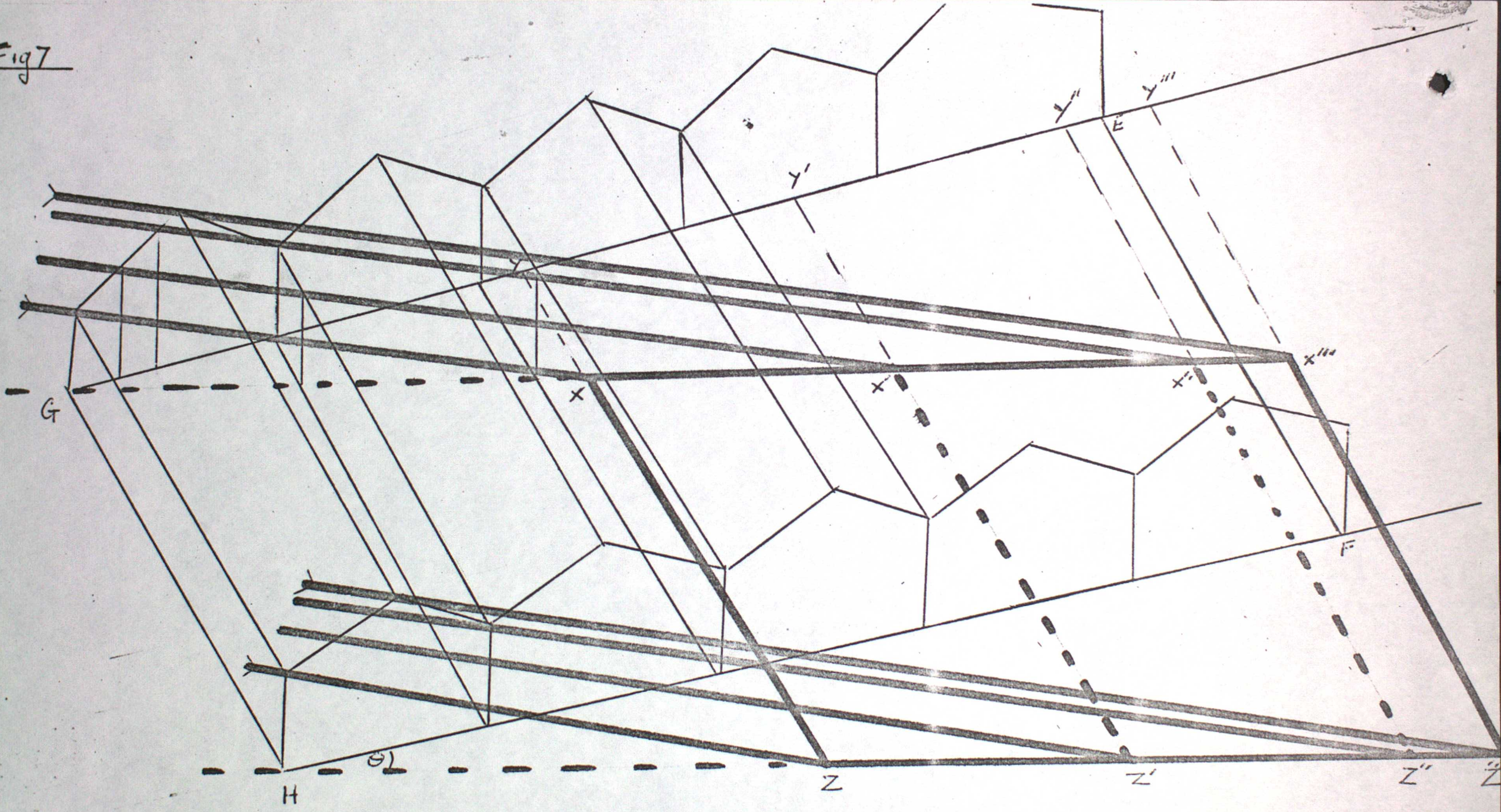

Y'''
Y''
E
Y'
X'''
X''
X'
X
F
G
H
θL
Z
Z'
Z''
Z'''

Fig 7 (a)

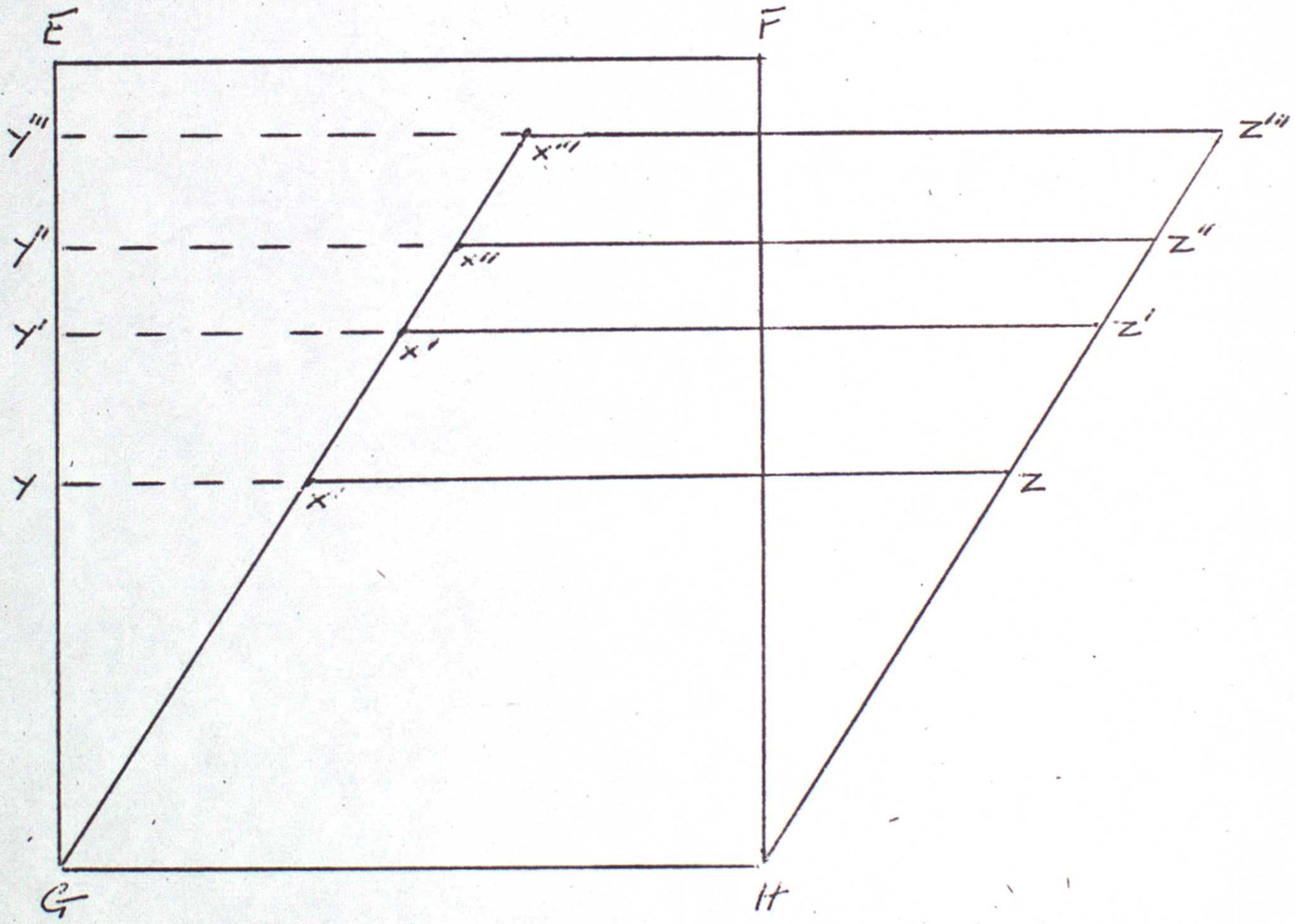

E
F
Y'''
Y''
Y'
Y
X'''
X''
X'
X
Z'''
Z''
Z'
Z
G
H

Fig. 8

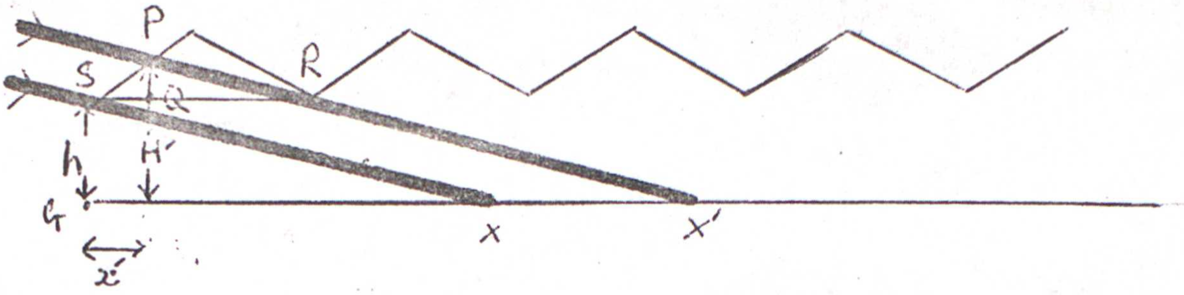

P
S
Q
R
h
H′
G
x′
X
X′

Fig. 9.

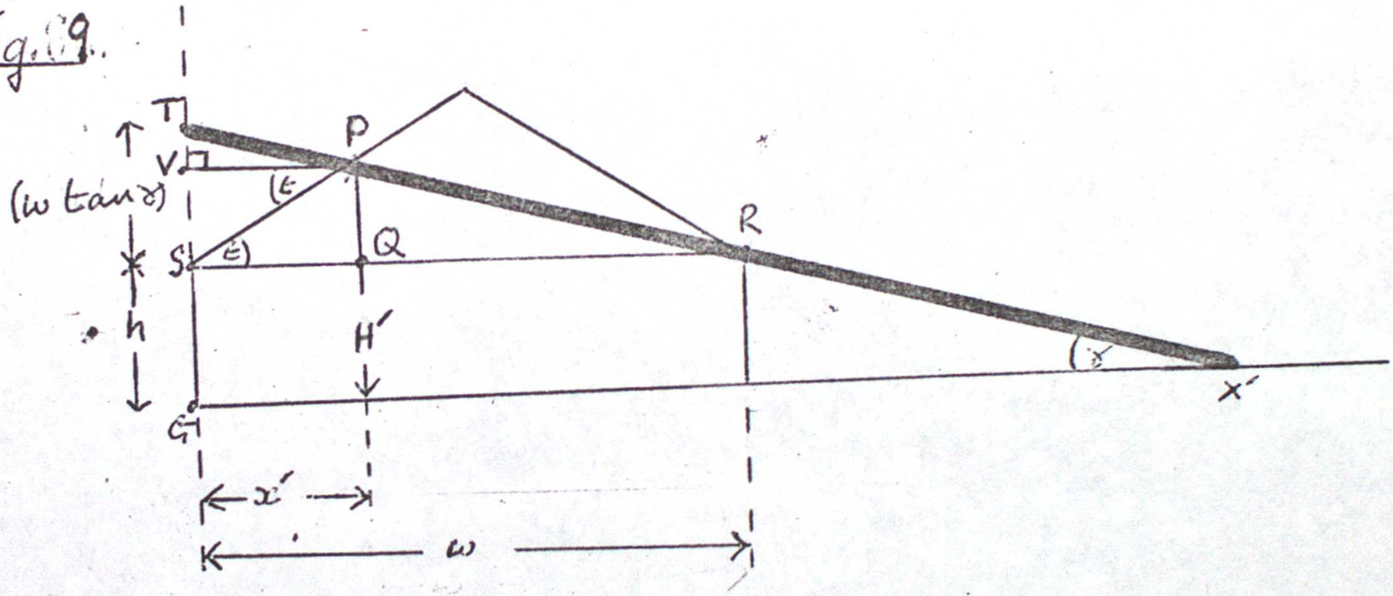

T
V
P
(w tan γ)
ε
ε
S
Q
R
h
H′
γ
X′
G
x′
w

Fig.10

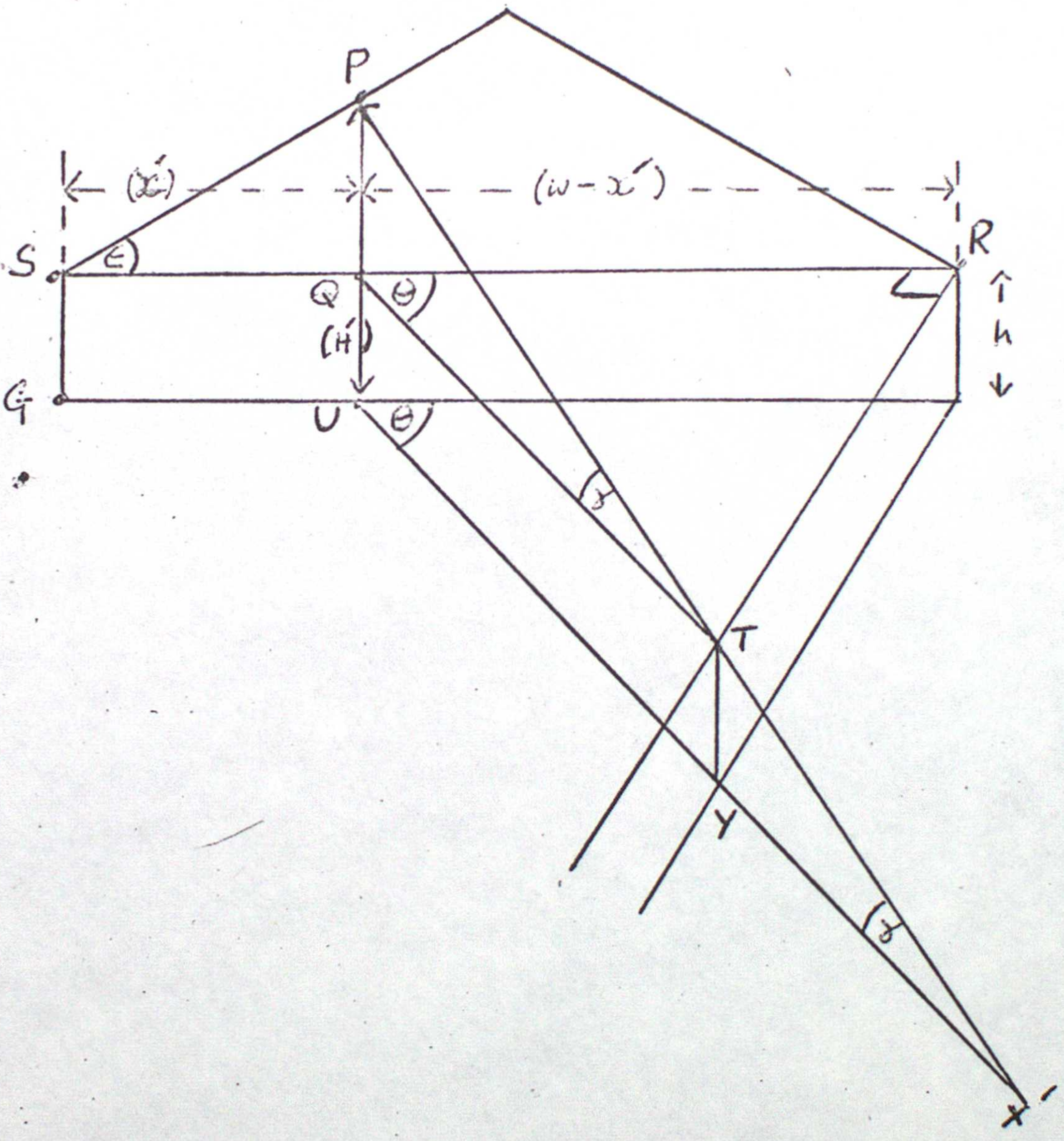
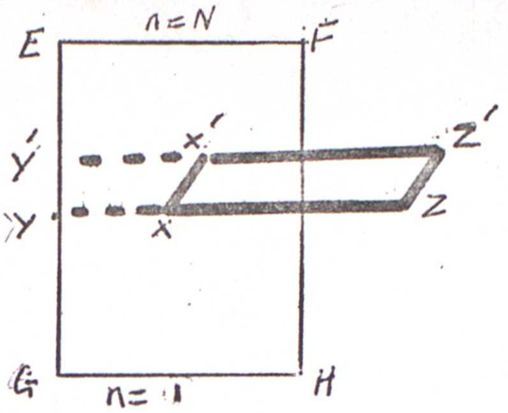
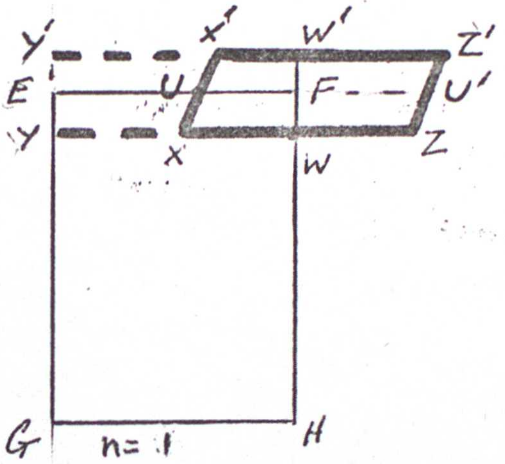

P
(x')
(w - x')
S
ε
R
Q
θ
h
(H')
G
U
θ
γ
T
Y
γ
X

Fig. 11

Case I


n=N
E
F
Y'
X'
Z'
Y
X
Z
G
n=1
H

Case II


Y'
X'
W'
Z'
E
U
F
U'
Y
X
W
Z
G
n=1
H

Case III

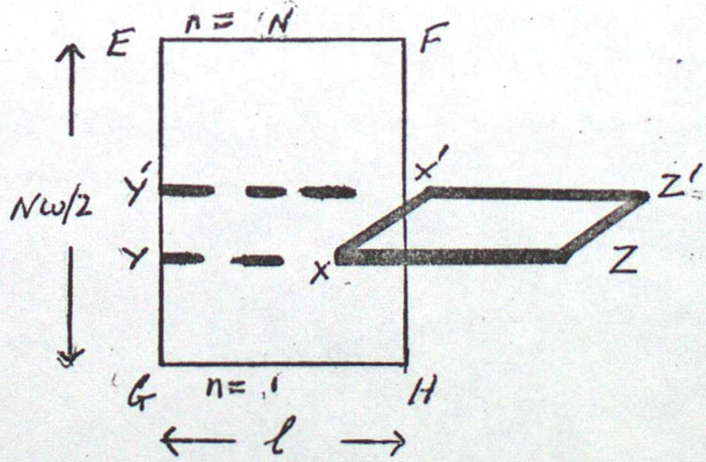

n=N
E
F
Nω/2
Y'
X'
Z'
Y
X
Z
G
n=1
H
l

Fig. 12

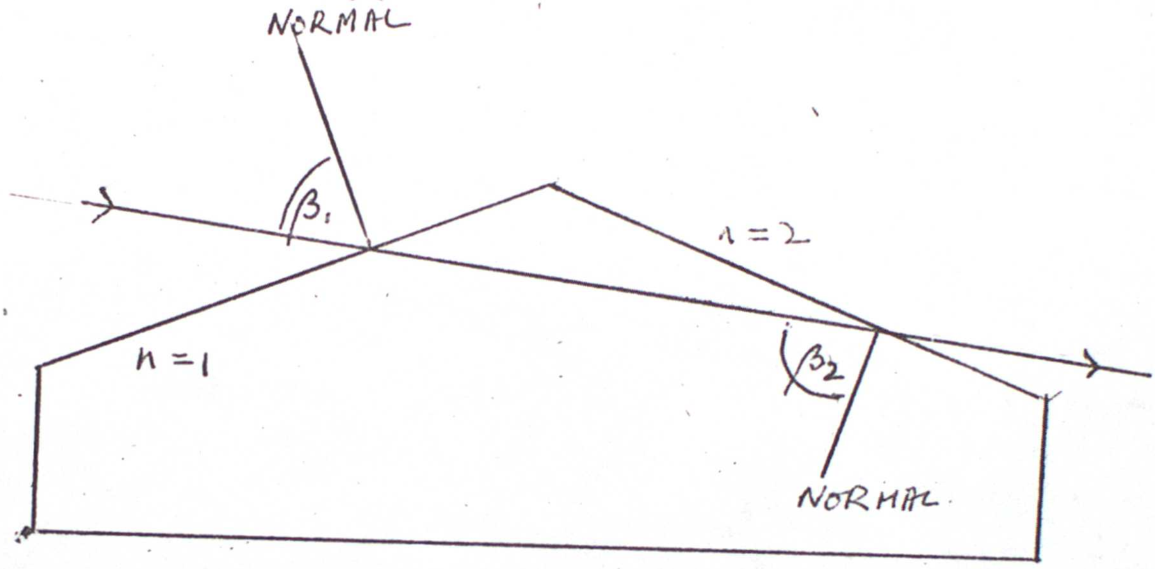
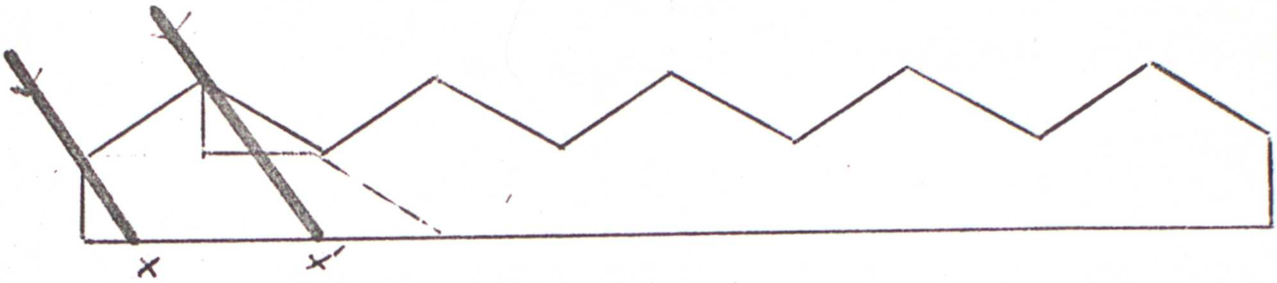
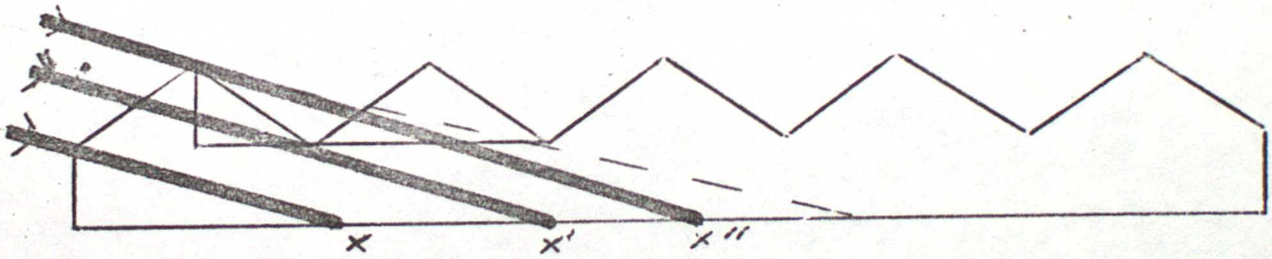

NORMAL
β₁
n = 2
n = 1
β₂
NORMAL

Fig. 13

(a) $\gamma > \epsilon$ ie $w/2(h'-h) \geq \cot\gamma$



(b) $3w/2(h'-h) \geq \cot\gamma > w/2(h'-h)$



(c) $5w/2(h'-h) \geq \cot\gamma > 3w/2(h'-h)$

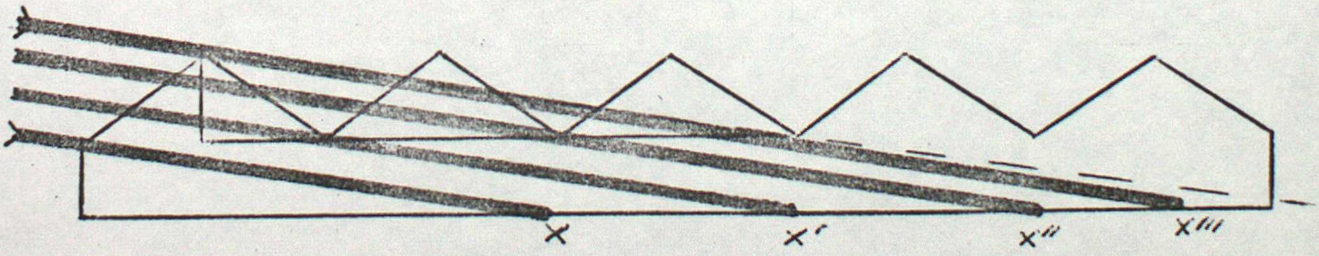
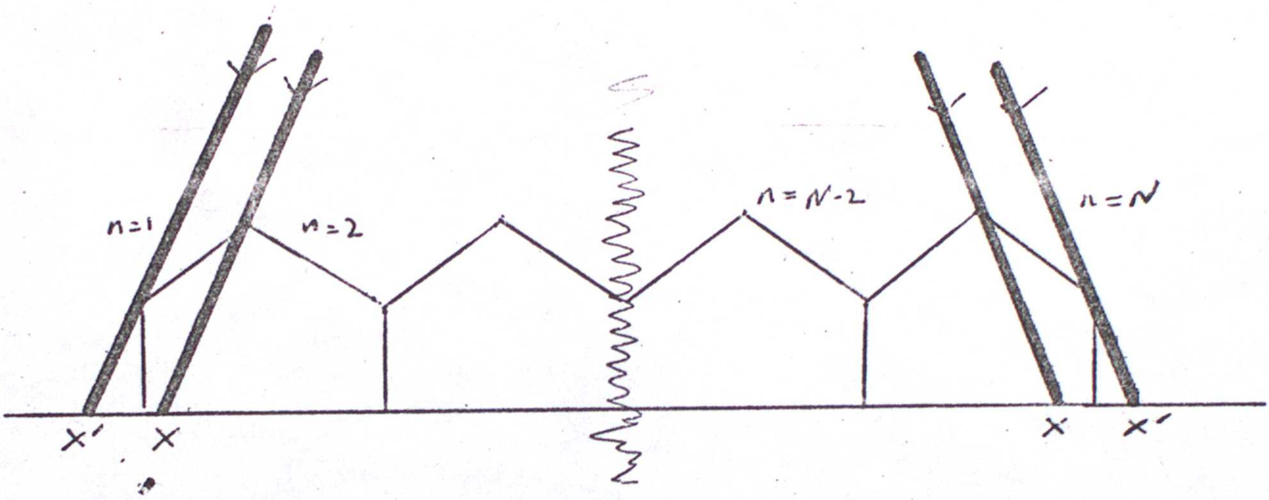


Fig 14



n=1
n=2
n=N-2
n=N
x'
x
x
x'

Fig 15

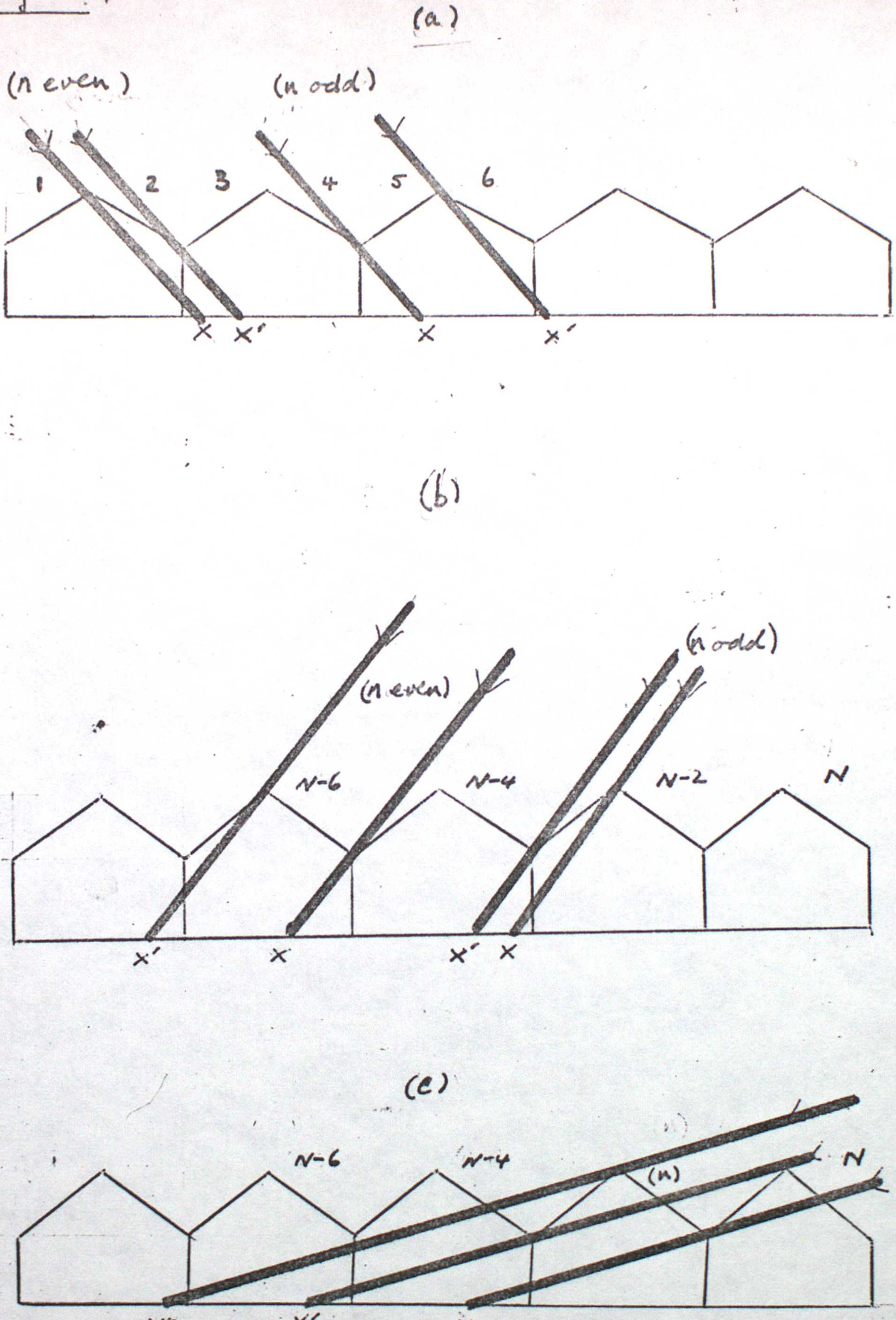

(a)
(n even)
(n odd)
1
2
3
4
5
6
X
X'
X
X'
(b)
(n even)
(n odd)
N-6
N-4
N-2
N
X'
X
X'
X
(c)
N-6
N-4
(n)
N
X''
X'
X

Fig. 16

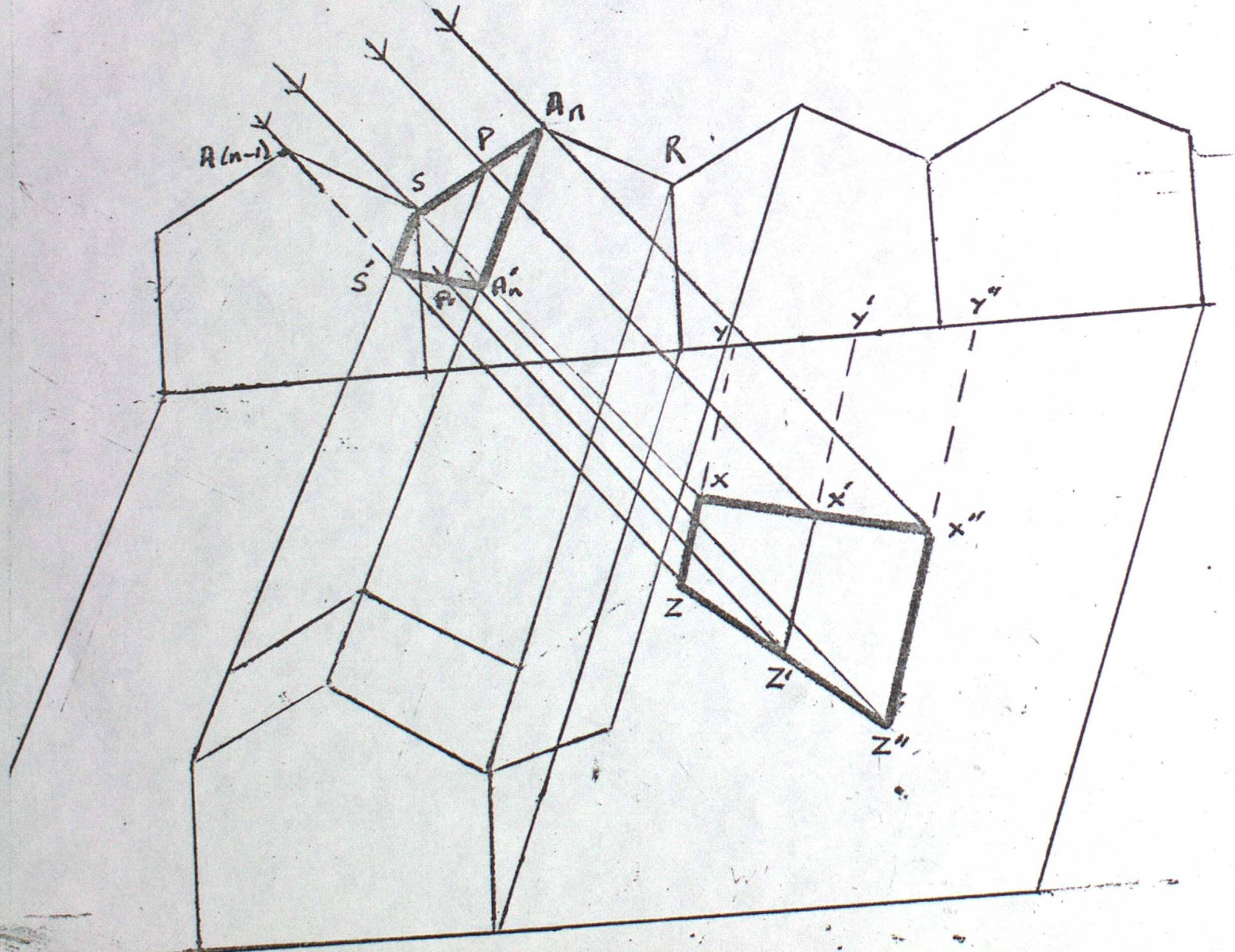

A(n-1)
S
P
An
R
S'
P'
A'n
Y
Y'
Y''
X
X'
X''
Z
Z'
Z''

Fig. 17

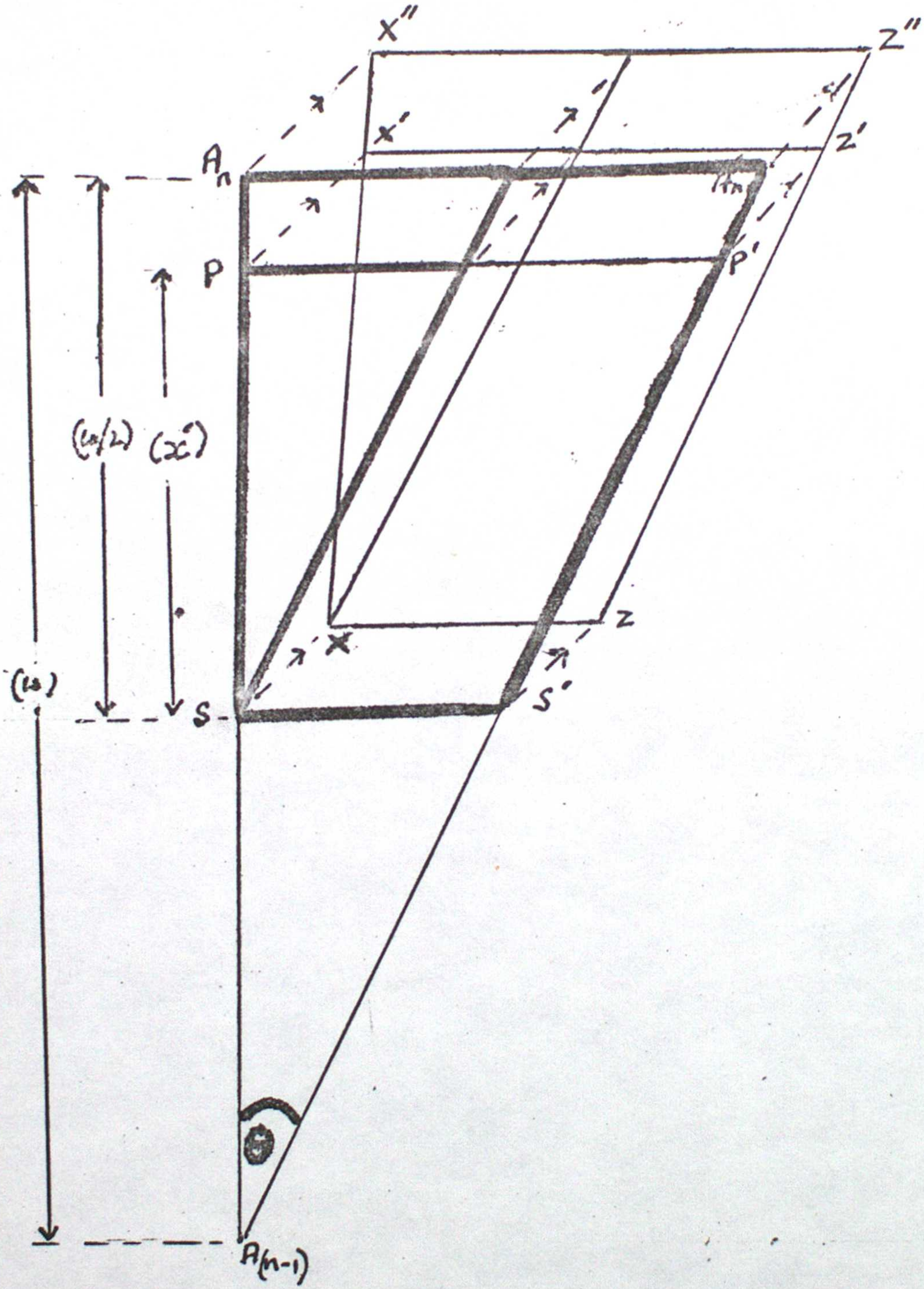
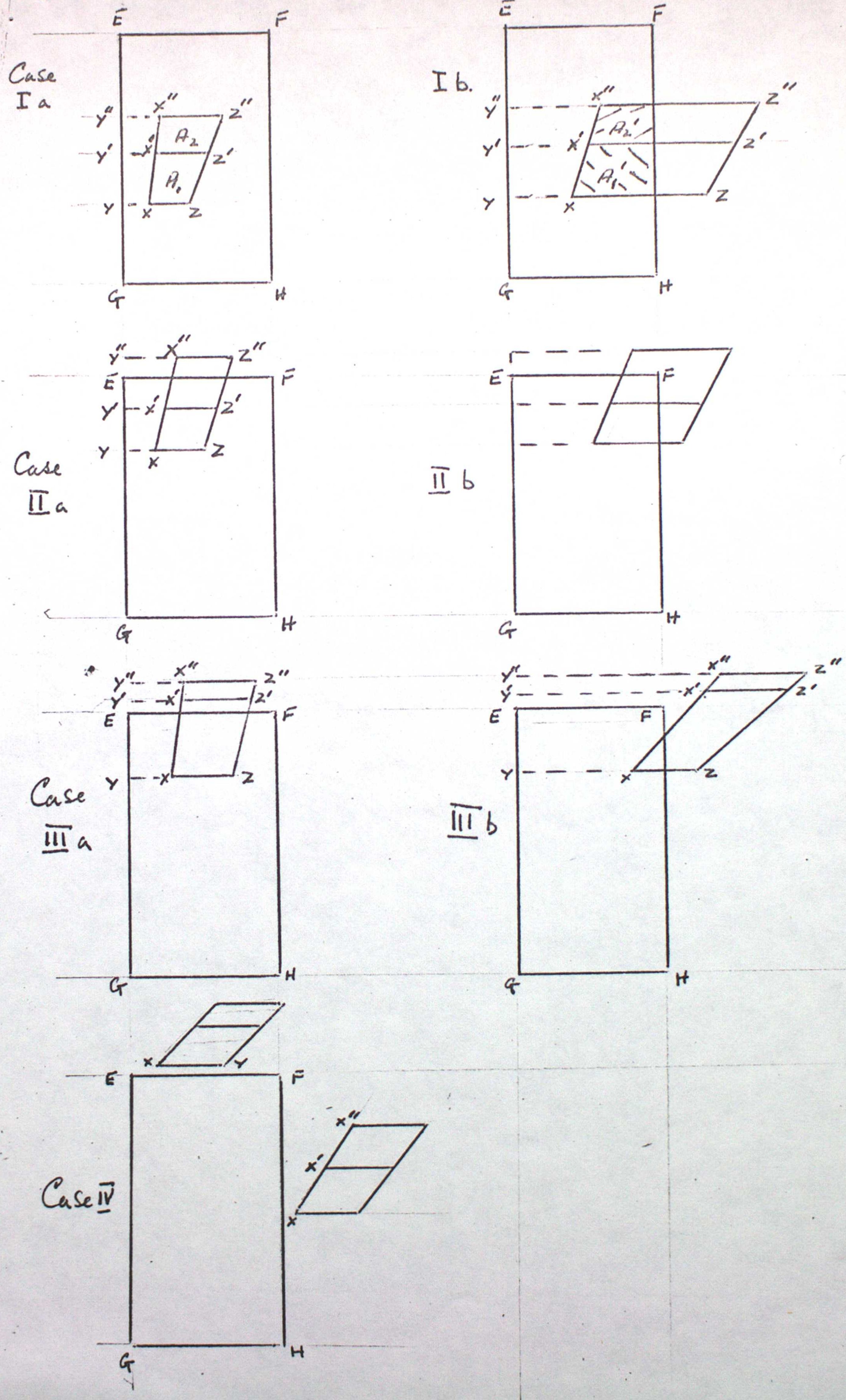

X"
Z"
X'
Z'
A_n
A_n
P
P'
(u/2)
(x')
(u)
X
Z
S
S'
θ
$A_{(n-1)}$

Fig. 18



Case Ia

Ib.

Case IIa

IIb

Case IIIa

IIIb

Case IV

E
F
G
H
X
Y
Z
X'
Y'
Z'
X''
Y''
Z''
A_1
A_2
A_1'
A_2'

Fig. 19

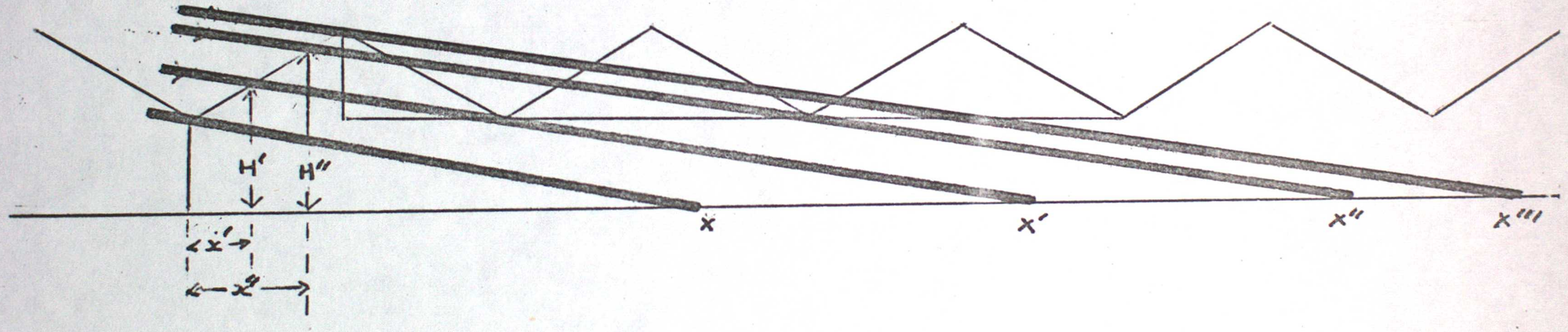


Fig. 20

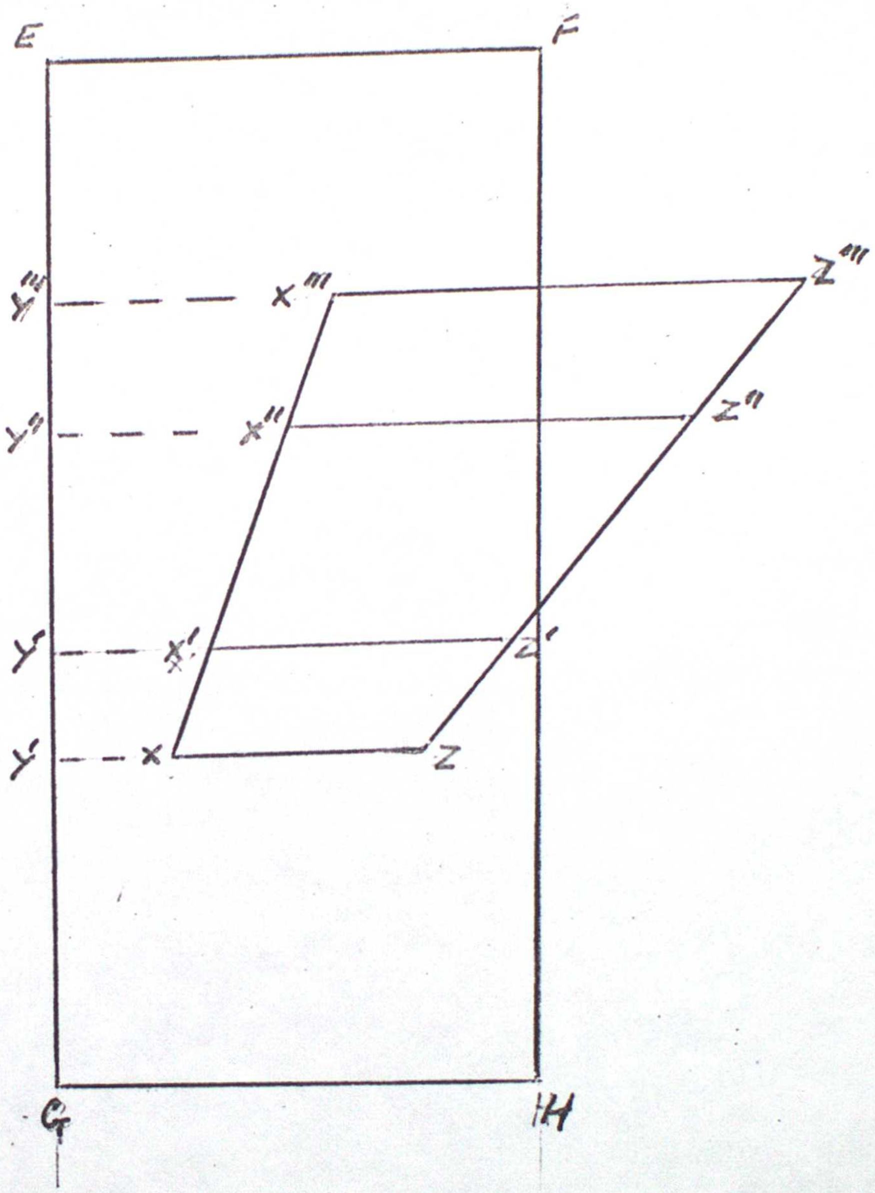

E
F
Y'''
X'''
Z'''
Y''
X''
Z''
Y'
X'
Z'
Y
X
Z
G
H

Fig.21

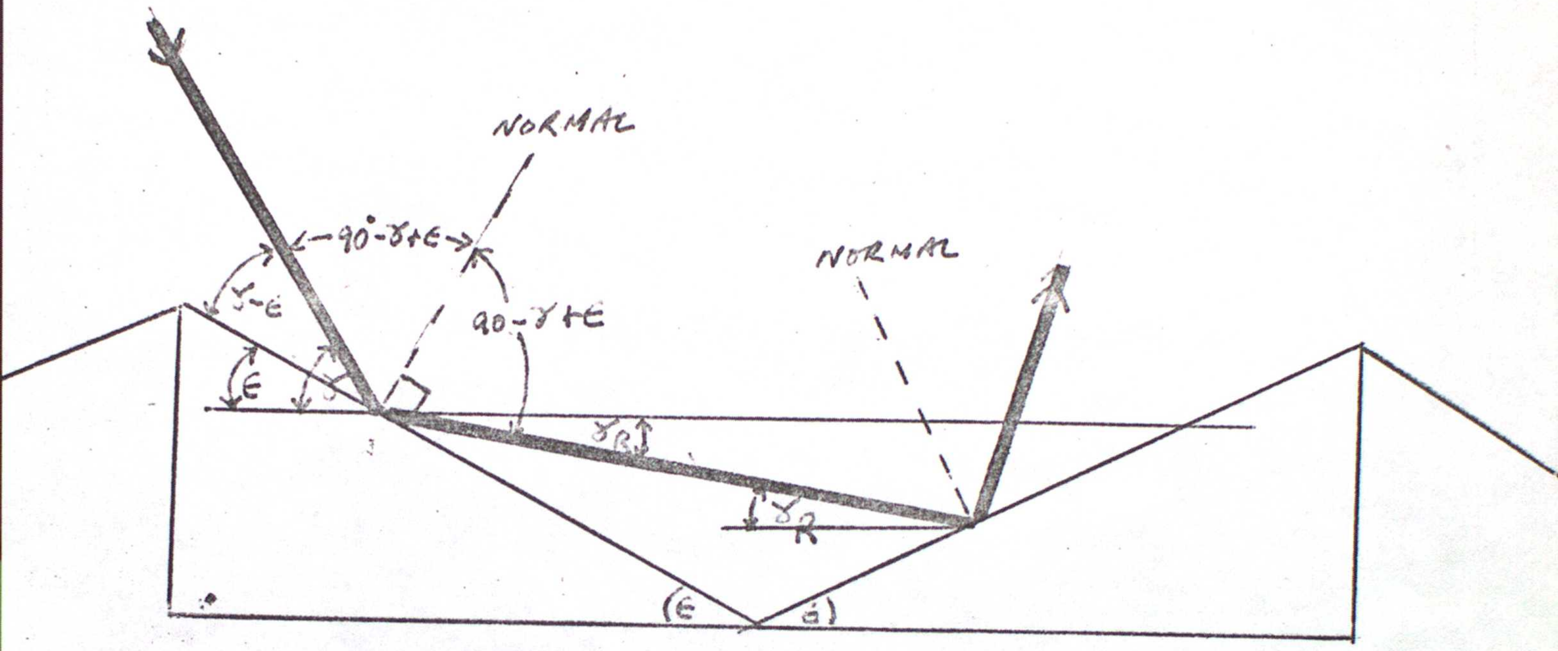
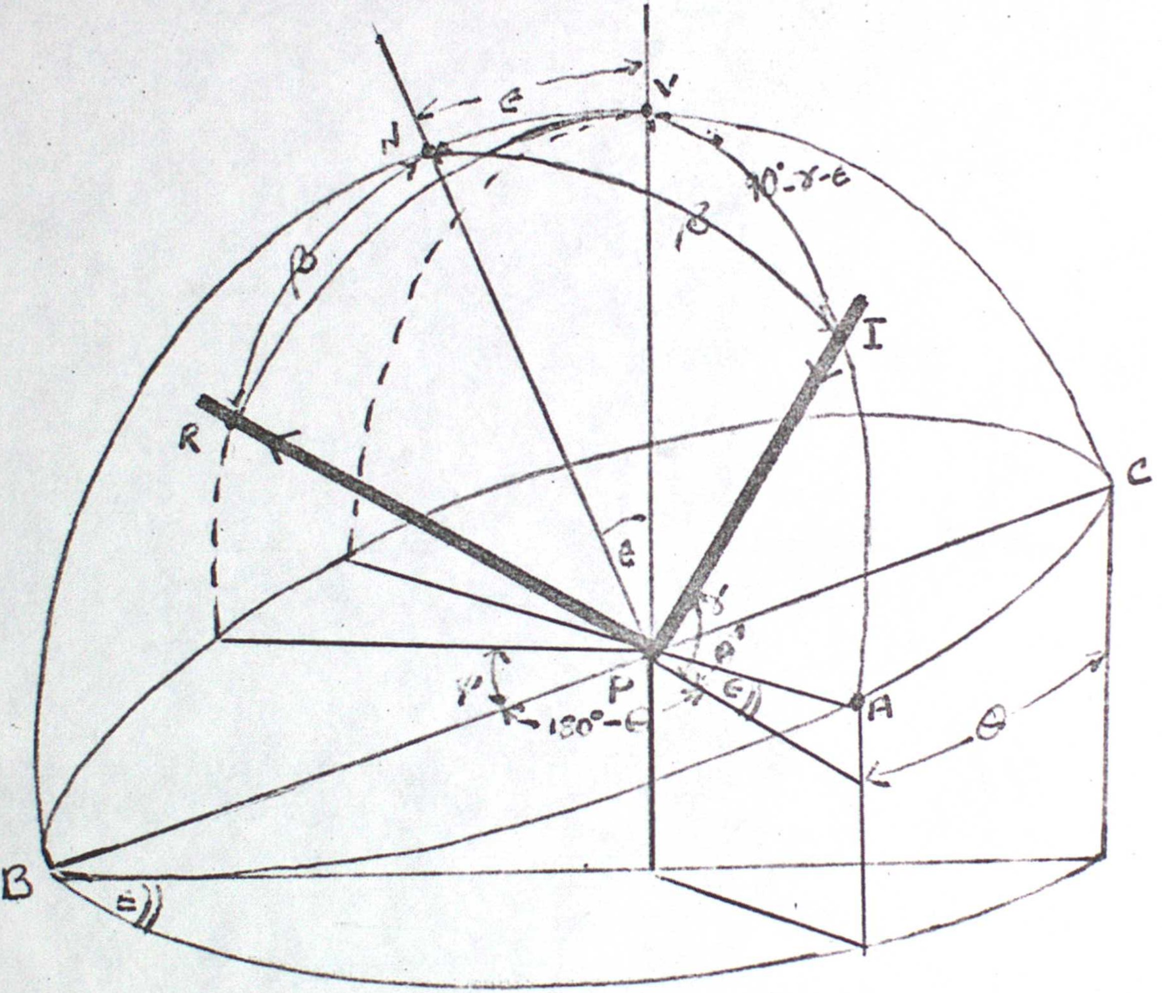

NORMAL
NORMAL
$90^\circ - \gamma + \epsilon$
$90 - \gamma + \epsilon$
$\gamma - \epsilon$
ϵ
γ
γ_R
γ_R
ϵ
α

Fig. 22.



N
V
ε
90°-γ-ε
β
γ
I
R
C
ε
γ
θ'
γ'
P
180°-θ
ε
A
θ
B
ε

Fig. 23

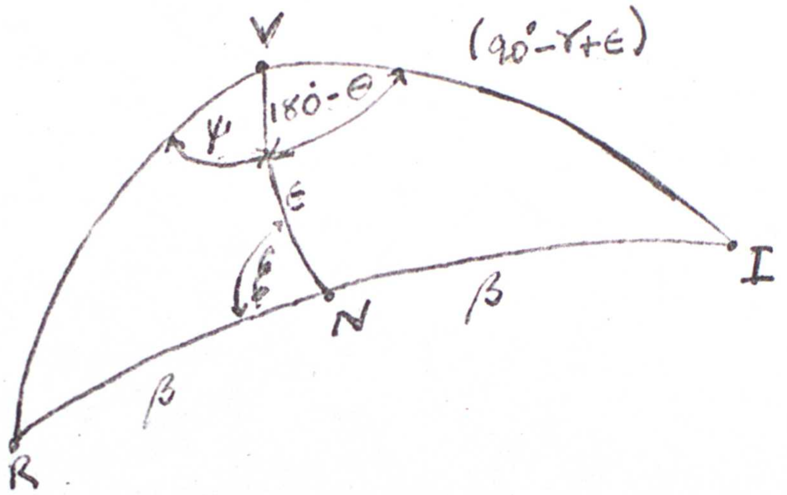

V
(90°-γ+ε)
180°-θ
ψ
ε
φ
N
I
β
β
R

Fig. 24

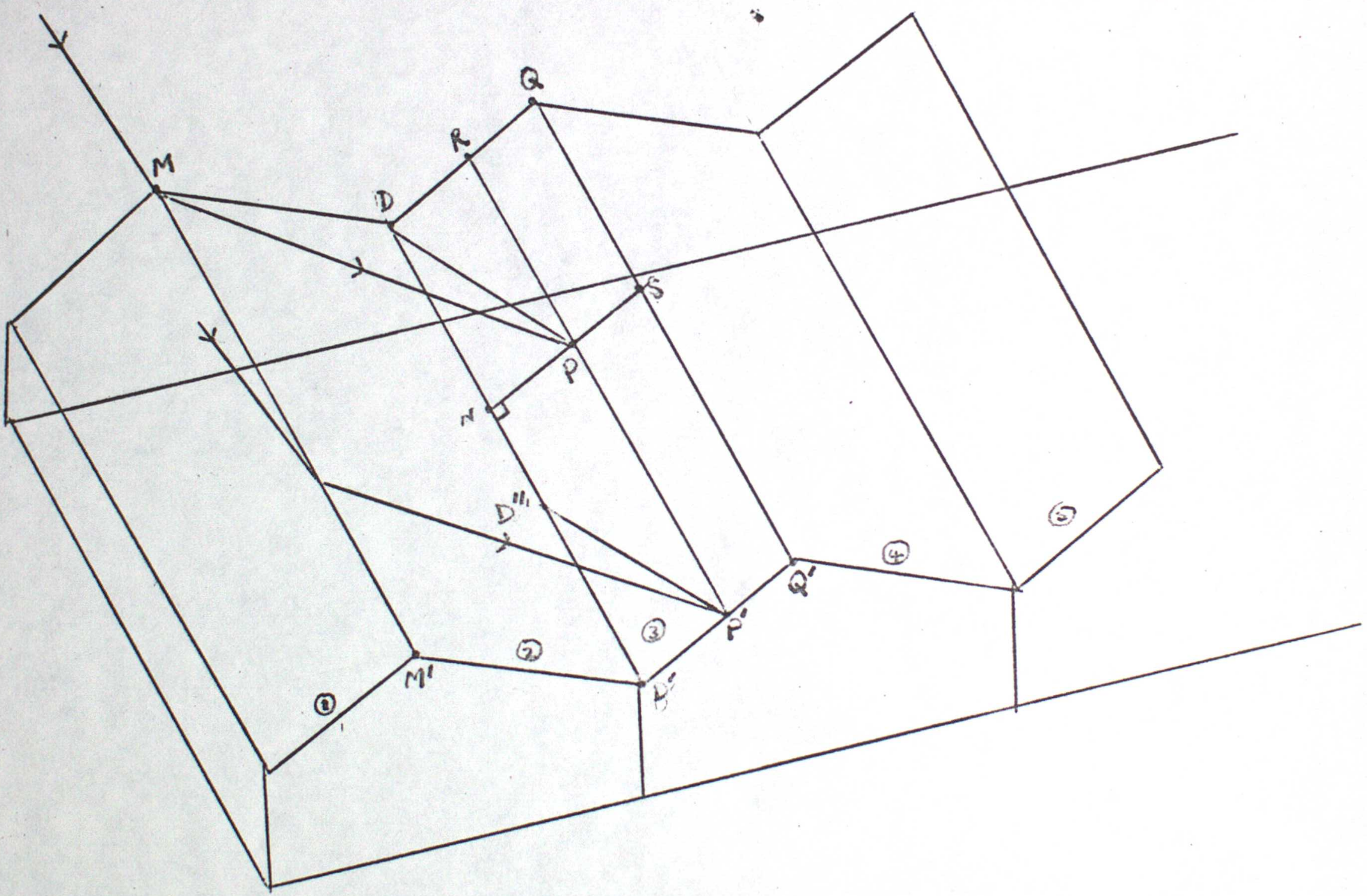
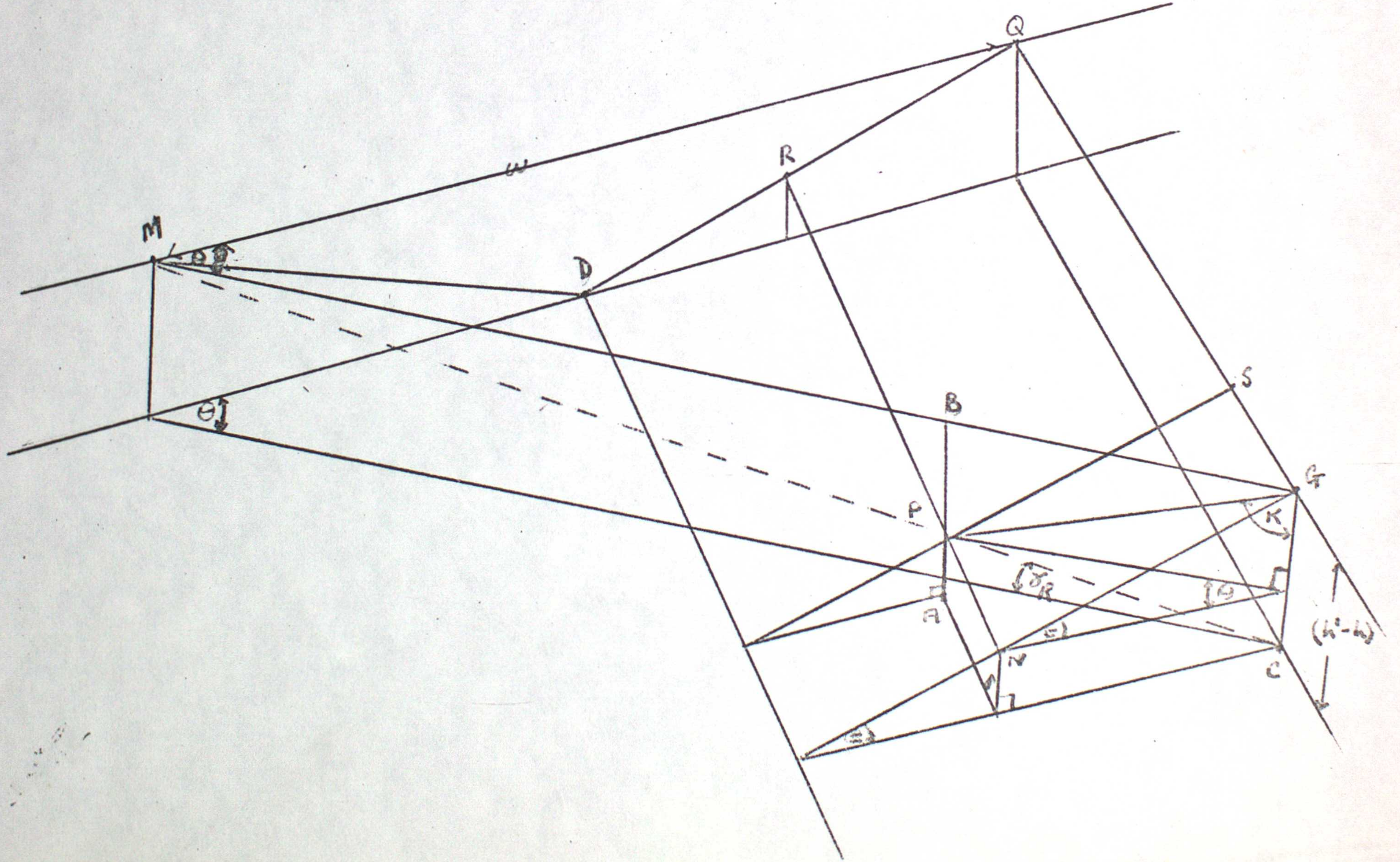

M
D
R
Q
S
P
N
D'''
M'
D'
P'
Q'
①
②
③
④
⑤

Fig 25


Q
R
w
M
θ
D
θ
B
S
G
K
P
δ_R
θ
A
ε
N
C
(h'-h)
ε